# ON THE CONSTRUCTION OF THE VAULTS OF THE MIDDLE AGES. 

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## INTRODUCTION.

IN the year 1568, Philibert de l'Orme published at Paris his treatise on Architecture. This is the first work in which the art of masonic projection is introduced, and it contains a very complete essay upon that subject, which our author denominates the art of describing " les traicts géométriques qui monstrent comme il fault tailler et coupper les pierres," and which in fact is the art of obtaining from the plans and designs of a proposed building the shape of the individual stones in a convenient form for the use of the mason. This first essay was followed by two works on the same subject by Maturin Jousse and Derand, which appeared nearly at the same time, namely, in the years 1642 and 1643. Other writers in France published upon this favourite subject, namely, Desargues in 1643, De la Rue in 1727, and Frezier in 1738. In fact, this has always been, and is now, a standard branch of architectural education in France, under the name of the "Coupe des pierres." It is joined with the corresponding art in carpentry, of which the first work was also written by Maturin Jousse, and published in 1627. In our own country this art has been rather neglected ; our only writers are Halfpenny, "Art of Sound Building," 1725, and Nicholson, whose various works on this subject are too well known to professional men to need enumeration.

Now although the merit of reducing this art to written laws and systems, and thus of giving it a place in the literature of science, belongs undoubtedly to De l'Orme and his immediate successors already enumerated, it must not be supposed that he was the inventor of the art. On the contrary, the necessity for it plainly arose very early in the history of masonry. Indeed, De l'Orme bever pretends to be the first inventor. His own language is that of a curious
pedant in his art, and he manifestly introduces many new methods and forms; but generally he talks of the matters which he is teaching as belonging to the ordinary practice of his age, which he is for the first time reducing to writing. It must be remembered, too, that the forms of the vaults which serve him for examples belong to the Italian style, and are very different from those of the Middle Ages ; so that granting to him the merit of applying and modifying the old Gothic methods to adapt them to these new forms, it is plain that much of originality is due to him. He, like all his contemporaries, imagined his own architecture to be a genuine restoration of the ancient classical styles; and therefore he speaks of the Gothic vaults as "voutes modernes que les maistres maçons ont accoustumé de faire aux églises et logis des grands seigneurs" (p. 107). "Auiourd'huy ceux qui ont quelque cognoissance de la vraye architecture, ne suinent plus ceste façon de voute, appellée entre les ouvriers La mode Françoise, laquelle véritablement je ne veux despriser, ains plustost confesser qu'on y a faict et pratiqué de fort bons traicts et difficiles." Frezier also, speaking of Gothic vaults, says with great truth, "Toutes ces naissances entrelassées, et les intersections des moulures demandoient une grande intelligence dans l'Art de la Coupe des pierres; d'où je conjecture, que c'est à l'architecture Gothique que nous devons rapporter l'origine, ou du moins l'adolescence de cet Art. Ma raison est qu'outre qu'il ne nous reste pas de Monumens antiques où il ait été mis en usage que pour des traits assez simples, c'est que dans l'énumération que Vitruve fait des connoissances nécessaires à un Architecte, il ne parle point de celle de la Coupe des Pierres; en effet, la noble simplicité de l'Architecture des Anciens n'exerçoit pas beaucoup le sçavoir-faire des appareilleurs, qui n’avoient presque que des voutes cylindriques ou sphériques à conduire." (t. 1, p. xvii.)

It becomes, therefore, a curious and interesting subject of inquiry to trace, from an examination of the structures themselves, what geometrical methods were really employed in setting out the work, and how the necessity for these methods gradually arose. Independently of the value of such investigations to the history of the science of construction, the knowledge of the methods actually employed would greatly assist us in the imitation of the works of each period. For the forms and proportions of every structure are so entirely dependent upon its construction and derived from it, that unless we thoroughly understand these constructions, and the methods and resources which governed and limited them, we shall never succeed in obtaining the master key to their principles, and instead of designing works in the style of any required age, we must content ourselves with merely copying them.

The following paper must be considered as an attempt to sketch out an investigation of this kind, and in offering it to the body of practical men who are assembled in this Institute, I am not without hope that some of them may be induced to collect facts and examples by which this investigation may be carried on and completed. For it will appear, as we proceed, that most of the facts required are of such a nature that they can only be derived from the existing buildings by the aid of scaffolding, minute measurement, and close observation, which it is not often in the power of mere travelling observers to obtain.

Now professional men are so commonly entrusted with the repairs or restoration of these old structures, that if they would take the opportunity of making the required observations in every case where scaffolds were erected about a building, and if such observations were transmitted to the Institute, a few years would suffice to bring together a body of examples from which general rules might be deduced. It is only by comparing many examples that this can be done, for general rules deduced from single instances are commonly worthless.

Next to complete buildings under repair, Ruins afford the most valuable information upon construction. The best instructor of all, perhaps, is a building which is being pulled down, but such opportunities are always to be regretted. In ordinary cases, the upper surfaces of the vaults are so often covered with courses of rubble and concrete, rubbish and filth, and the lower surfaces witb whitewash and paint, that when every facility has been obtained for examination, the jointing of the masonry and actual construction of the vault will still remain an unfathomable mystery.

## Section I.-On the General Construction of the Vaults.

The ribbed vault of the Middle Ages, which is the subject of the present paper, differs entirely from the vaults of the Romans. It consists, as is well known, of a framework of ribs or stone arches, upon which the real vaults or actual coverings of the apartment rest. These vaults are usually constructed of a lighter material and with rougher workmanship than the ribs upon which they rest, and between which they constitute, in fact, a kind of thin pannel.

The construction of these ribs offers no difficulty in stone-cutting, each being, separately considered, a simple arch; nevertheless the forms of its voussoirs
can scarcely have been obtained without tracing on the ground the full-sized figure of this arch, from which to obtain the face moulds of the voussoirs.

However, in the Norman ribbed vaults the arches are so roughly constructed that it may be supposed that a small portion of the arch only was described and a single face mould employed for the whole, and that the joints of the voussoirs were not divided upon a complete full-sized drawing of the arch. We shall see, however, that this full-sized drawing was very soon required in the early English period.

Some of the Norman vaults are plain waggon vaults of rubble work as in the nave of the chapel at the Tower, or groined vaults of rubble as in the side aisles; but it is unnecessary to dwell upon these very early specimens, in which no difficulties either of stone cutting or of curvature are yet intro. duced ${ }^{1}$. It is only where the groins of the vault are supported by ribs that the branching of these ribs in various directions from one abacus, and the different spans of the transverse and diagonal ribs, introduce difficulties; which were at first, as might be expected, somewhat clumsily botched over or evaded, and which afterwards led to the invention of the geometrical system, the rise and progress of which it is my purpose to trace. In Norman ribbed vaults, however, each rib springs independently from the abacus, and. appears to have been erected without any reference to the management of the incumbent vaults. The latter are formed of rubble and irregular work, and may probably, in some cases, have been packed together without centering ${ }^{2}$, or at least with a very rude framework; and to accommodate the curvatures of the backs of the ribs, especially near the abacus where the ribs approach, these ribs are backed out where necessary with irregular stones and rubble work, There is an excellent example of this kind of vault in some apartments on the west side of the south transept of Peterborough cathedral. These have been long employed as mere workshops, and the surface of the vaults being denuded of plaster, if it ever had any, its construction is plainly shown. The compartments are covered with a cross-ribbed vault, of which the diagonal arches are semicircular, and the transverse pointed. The span of the latter is

[^0]bout thirteen feet. The voussoirs of the diagonal ribs are twelve inches square the transverse section, and backed up, as already described, near the springing, so as to throw the spring of the vaults to a considerable height above the abacus. There are many other curious irregularities and peculiar forms in this vault, which is well worth examining.

Another good example is in the castle at Newcastle-upon-Tyne, in a large vaulted apartment about twenty-seven feet by twenty ${ }^{3}$. This has a central pillar, from which spring four transverse arches and four diagonal arches. Upon these the vaults rest. This is a very curious specimen, for every pair of these arches has a different span: that of the two transverse arches upon the long diameter of the room being twelve feet three inches each; of the two shorter transverse arches, nine feet ; and of the diagonal arches, about fifteen feet nine inches.

All these arches are semicircular, and their crowns are all placed at the same level, to allow for which they are made to spring from different levels. Fig. 1 is a sketch of the springing of these arches from the central pillar. $C D$ is one of the small transverse arches, which having the least span, springs from the highest point $c . ~ A B$ is one of the greater transverse arches, springing from a below the level of c, and Ef, кl, eh, are diagonal arches, whose span being the greatest, spring from still lower points, e, к, and g, so that the crowns of all these arches rise to the same level. With respect to the voussoirs, it will be seen that although the backs or extrados of the voussoirs which constitute the arches are concentric to the soffits of these
 arches, yet these are backed up by a rude wall of rubble work, as at $m$, N and $P_{\text {s }}$ upon which the rough vaults are supported, and this is necessary to get rid of the difficulty of resting a vaulting surface upon arches whose curvature and level at the springing are so different.

Fig. 2 is another example, which perhaps shows the nature of this difficulty more clearly. This is a portion of a vault from the ruins of Finchale Priory, in the county of Durham. AB CD are two ribs or arches which spring with very different curvatures from a single pillar. The vault is in this case formed of long thin stones, packed one upon the other, as shown at efg. And as

[^1]the different curvatures of the arches throw the surface at $f$ much higher than that at $e$, the difference of level is made up by courses of rubble upon the lowest extrados of the two only between $e$ and $m$, which are gradually thinned off at the latter point, and enable the stones of the vault to be laid straight from one arch to the other.

From these and many other examples it appears that in this early stage of rib vaulting the arches or ribs consist of independent and separate voussoirs down to the level course from which they spring, as in fig. 2,


FIG. 2. where a and c, $a$ and $c$, are separate stones roughly jointed at the back, instead of being each got out of a single stone, as in the subsequent structures. Also the back or extrados of these ribs is concentric with the soffit, and thus formed without reference to the arrangement of the incumbent vault, which was plainly a subsequent and separate consideration; the ribs being, after their erection, backed out at the springing so as to accommodate the curvature of their extrados to the reception of the vault.

The vaults of the choir at Canterbury have in like manner their ribs formed of separate stones, from the abacus upwards, and backed up where necessary; the voussoirs of the ribs are also very small and numerous: I counted about a hundred in one transverse rib of the north-eastern transept in a span of thirty feet. These ribs are very richly moulded, but the workmanship is exceedingly rude.

To this rough construction of the spandrel succeeds all at once a more artificial structure, bespeaking a great advance in the art of masonry; and it is remarkable that this new construction once introduced remains with very slight change to the very latest period of rib vaulting.

Fig. 3 is a diagram to illustrate this construction, and is principally derived from the south transept of Westminster Abbey, but is not drawn to scale. The left hand half of this diagram represents a portion of the vault in perspective, including the entire spandrel
 solid (if I may be allowed the expression), which is contained by the two semi-
diagonal ribs ADAE and the wall. ABC is the transverse rib, and on the right fand of the diagram the vertical section through this rib is exhibited. G f is the string moulding upon which the clerestory windows rest, and the arch which contains these windows springs from LK at a considerable height above the springing, A, of the vault ribs. This is a very universal arrangement of clerescory vaults, and is productive of great beauty and convenience, but it leads to some difficulty in the form and arrangement of the vaulting surface akfd, for as this is contained between two arches or ribs, AD KF, which spring from different levels, it follows that this surface must be skewed back at $\kappa$ in a very peculiar manner. This is shown by the perspective. The junction of the solid mass, AKQ, with the clerestory wall is therefore bounded by parallel vertical lines, one of which is a $K$, and this mass is always built of solid masonry bonded into the wall and forming a part of it ${ }^{4}$. It is from the level of kq that the real rib and pannel ${ }^{5}$ construction of the vault begins, for separate ribs are erected upon the surface of this solid, and connected by vaults of a light material. From below, however, if the vault be painted and decorated, this change of construction at кQ is disguised, or, in other words, the decorative construction of the vault exhibits the rib and pannel from the abacus a upwards, but the mechanical construction is of solid masonry from $A$ to $Q$, and of rib and pannel work only above this level. The point Q of this change of construction Is commonly at about half the vertical height of the arch, as shown in the drawing, but is not necessarily at the same level as the impost k of the clerestory rib к f.

The peculiar construction of the solid mass akQ is better shown in the section at cmn. The ribs of the vault converging downwards to c, their mouldings become entangled as it were, in a manner that will be subsequently explained. At a point m, about half way up the solid, they are, however, freed from each other, and separated by the divergence of the ribs. Now, between band $m$ the solid is built of horizontal courses of masonry, generally each

[^2]of a single stone, and its level beds cut the curved mouldings obliquely in front. Above $m$ the ribs are each built separately of voussoirs, having their beds properly inclined to meet the axis of curvature P of the rib, and these ribs are backed and united by solid masonry which connects them with the wall, and which appearing between the ribs seems to be a portion of the light vaulting surface, such as is really employed higher up. From the upper surface $n$ of the solid, each rib $\mathrm{N}_{\mathrm{B}}$ is still built as from m to N with voussoirs, but upon these ribs rests the light thin vault or pannel, shown in section on the right, and in perspective on the left of the diagram.

It is remarkable that the courses of the vaults are not laid level, but are in most cases made to incline downwards upon the diagonal rib.

Thus in $f g$. 3 the ridge FD is level, and also the ridge DBE ; but the courses of the vaults incline considerably downwards from FK and from QB towards the diagonal rib AD. These courses, in Westminster transepts, are of a light-coloured stone, probably chalk, interrupted, at regular intervals, by a course of a darker stone; and the ridge FD, which has no rib, is also formed entirely of this darker stone, laid in the serrated manner shown by the drawing. The dark courses are rather broader than the light ones, and there are four or five courses of the light between each of the dark. The surface EKD is also slightly concave or domical, and may therefore have been laid without any centering, since each course would support itself ${ }^{6}$. These peculiarities may all be found with some variations in other vaults of the same age.

What might have been the reason for this downward inclination of the courses it is not easy to say, but it is very common, especially in the earlier examples. Some have supposed it to have arisen from the courses having been laid to meet the bounding ribs AK F A D at respectively equal distances from their springing, which would certainly produce the effect in question, since the diagonal rib is so much longer than the others, but the downward inclination is greater than that which would arise from this cause. In some examples the slope seems to be derived from the courses having been laid so as to meet the diagonal rib at right angles. The perspective effect which arises from the arrangement is curious, for the vaulting surfaces $A D B A B E$ are really very nearly coincident with a single surface extended from $A^{\prime} D$ to $A E$, or in other words, a horizontal rod passed upwards along the backs of the ribs AD AE would very nearly touch the two vaulting surfaces and the back of the rib $A$. But the effect of the inclination of the courses is to make the rib $A B$

[^3]appear in perspective as if prominent downwards from such a surface, and consequently gives to the entire solid spandrel FDEA the effect of a kind of fan vault or vault with a polygonal horizontal section.

Above the vaults are commonly laid a thick irregular course of rubble work, which again is also often covered with a coat of a kind of concrete. The vaults of Westminster Abbey, with the exception of the western compartments, those of Exeter, Winchester, Hereford (with the exception of the south transept and tower), Wells, Ely, Redcliff Church, Bristol Cathedral, and many jthers, are thus covered. These upper coverings appear to have been abandoned in the later periods, but not universally. Those of the western compartments of Ely choir seem to have been subsequently picked off, perhaps by Essex, to lighten the vault.

But the vaults of the western compartments of Westminster, and of the south transept and tower of Hereford, are left bare on the upper surface, and these vaults, instead of being built with small brick-like stones, are composed of long thin slabs. Also the ribs themselves are, in some later examples, formed of a few long bar-shaped voussoirs instead of the small and numerous pieces of the earlier examples. Thus in the transept of Westminster, н в consists of thirteen or fourteen stones, but at the west end of the nave, of six only.

The employment of the solid mass of masonry akq enables the ribs to approach more closely at the springing, and also reduces the actual span of the wault, for $Q \mathrm{~N}$ is the real span of the vauli, instead of A C , which is the apparent or decorative span. Thus about oue-sixth of the span is saved.

The early ribs are formed as in fig. 4 (from St. Saviour's Church, Southwark), the vaulting surface resting only on their backs; but the later ribs are rebated for the reception of the vaulting surface, as shown in fig. 14, by which greater depth and strength is given to them Without necessarily increasing their projection from the surface of the vault.

To return to the proper subject of this paper, namely, the evidences of a geometrical method in setting out the work: The solid spandrel from c to m, fig. 3, has been said to consist of level courses of masonry, and to contain all that portion of the vault in which the mouldings are entangled and partly concealed by the approximation of the ribs. Now two ways may be conceived in Which these mouldings may have been worked out of the stone. The spandrel may have been built solid, and after the ribs $\mathrm{m} N$ were set up, the mouldings of the entangled ribs from m to c may have been worked gradually downwards,
first in block, and afterwards in detail, and in this way the various interlacings and interpenetrations of the mouldings would develope themselves as thig workmen proceeded.

The other way would be to project geometrically upon each bed of the stones which constitute the spandrel from c to $m$ the mouldings of each rib in its proper position, and thus would be shown which mouldings remained prominent and which were covered by others; and that this method was employed I shall shew from examples. Of course a perfect existing building affords no opportunity of examining the beds of its stones, but if the beds of a newly pulled down structure be examined by carefully scaling off the mortar, the mason's lines will be found to remain as freshly upon the surfaces as when they were first set up, and from these it will be seen that a complete geometrical method was used to obtain the intersections of the mouldings.

Fig. 6 is a plan of one of the spandrel stones, taken down from the side aisle vault of St. Saviour's Church, Southwark, in the course of its demolition, in 1839. This drawing is reduced to one-eighth of the original, and the lines and narks which it exhibits are carefully copied from those that were found upon the surface or bed. Upon this were traced lines parallel to the direction of the
 wall and of the several ribs of the vault respectively, as FG in the direction of the wall, A в for the transverse rib, AC and DE for the diagonal ribs. The vault to which this spandrel belonged was of an irregular plan, four-sided, but having each side of a different length; to accommodate which, one of the diagonal ribs DE is made to spring from a point a little removed from the intersection a of the other Ac with the transverse rib a в.

Upon each of these lines the profile, or as much as is required of it, of each rib is traced, evidently by means of a templet, that is, of a pattern of the rib cut out in some thin material, which was held down in its proper place upon the stone while its outline was scratched or cut by carrying some sharp-pointed instrument round its edge. Thus the first traced was manifestly the complete profile $a b$ в $e f c d$ of the transverse rib. Next, on the left hand side is the profile $b c g k$ of the diagonal rib, and this by its projection obscures and renders unnecessary a portion $b a$ of the former rib; and that this profile was
traced subsequently to the former one is plain by the omission of all that is covered by it to the right of $b$. The wall-rib $g$ F was next traced, and this obscures the portion $g$ K of the former rib. In like manner, on the right hand side, the profile $c \mathrm{E}$ was traced, obscuring the piece $c d$ of the first rib. At a the stone is broken. The mouldings that were thus shown to project appear to have been then completely worked down to the outline thus obtained, excepting at one or two places, such as at $e$ and $f$, where the trace of the templet still remains within the real edge of the bed.

The compartments thus vaulted in St. Saviour's side aisles were not rectaugles but irregular trapeziums, and the diagonal ribs were slightly twisted upon the plan, as shown with some exaggeration in fig. 8. I have seen this in some other examples, but whether it results from design, with a view to dispose the branching ribs to better advantage, or from bad workmanship in not setting


FIG. 8. out the ribs in the tas de charge or solid block at the proper angles, and therefore making it necessary to warp their directions to enable them to meet at the crown, I am unable to decide.

To draw these figures upon the beds of the stones, it is necessary to know in each case the quantity of projection of every rib, or, in other words, the points C $\quad$ e to which the ribs extend at each joint, and against which the front of the pattern or templet must be placed before its outline can be traced. These are, however, so easily and obviously obtained by drawing on the full scale the elevation of each rib in its own plane, with the joints inserted after the manner of см, fig. 3, that we may assume that this was the method employed.

A diagram of this kind is to be found both on the upper and lower bed of each stone, and the lines a, D, \&c., which extend to the outsides of the stones, are scored vertically down the back for the purpose of making the two diagrams coincide. I may add, that every stone which I examined that had appertained to the spandrels exhibited similar lines and traces of mouldings; and as the pulling down of the buildings was proceeding during my examination, I had the opportunity of seeing the stones in their fresh state, and of myself scaling off the mortar.

The mouldings of these specimens are sufficient to show that they belong to an early period of the early English style. Fig. 7 is an example of a similar kind belonging to the Perpendicular period, and taken from one of the spandrels of a complex vault which formerly covered the extreme north-western compartment of the nave of Canterbury Cathedral, the lower story of the so
called Lanfranc's tower, which becoming ruinous, was taken down a few years since, and is now replaced by a modern copy of the south-western tower. The stones of the original vault, however, were carefully deposited in the nave of the cathedral and in the yard at the time of my visit, and I found the surfaces of these spandrel stones covered by lines and profiles of mouldings similar to those already described from St. Saviour's, showing that the same method had continued in use from the period of its first introduction.

The number of ribs that spring from each angle of this vault being seven, including the two wall ribs, made it necessary to employ two stones in each of its upper courses at least, and accordingly the bed represented in fig. 7, being only a portion of the entire spandrel, contains but four of these ribs. I have selected this one out of many others that I copied, because it is evident that a set of lines first drawn upon it were rejected because the stone was not large enough to contain the ribs, and another set was subsequently drawn and actually employed, which gives a somewhat additional interest to this example.
a $b$ ag are the rejected lines, and $d$ a portion of a profile of a rib belonging to $A$ G. BD BE BF BG are the true lines, drawn each parallel to its own rib, as shown by the plan of the vault. The profile D $m$ appears to have been the first drawn, then $n$ ep, then $q$ F, and so on. The marks at $s, t, i, \& c$. are
 apparently for the purpose of distinguishing the true lines from the false ones. The average thickness of the courses in this spandrel is about ten inches.

As the beds of the stones are horizontal in all these cases, the upper beds necessarily cut the mouldings of the ribs at an acute angle or feather edge, and this sometimes occasions the edges of the stone to fly off. On the uppermost surface of these stones (as at m, fig. 3). every rib has its own slant bed provided and worked square to the plane of each elevation, so as to give a firm footing to the separate ribs that all start from this upper stone.

One consequence of thus allowing the bed of the stone to cut the arch line obliquely is, that the templet or pattern from which the mouldings are traced upon the bed becomes too short in the oblique section; for (fig. 5) if $m n s p t$ be the upper stone, $m n$ the bed from which the detached ribs start, and if
$m n$ be the transverse depth of the mouldings, then it is evident that on the lower bed $p t$ the depth of the same mouldings $p r$ will be considerably greater on account of the obliquity of the section. Now in the examples that I have examined, I found that the same templet had been employed to trace the mouldings upon $m n$, upon $p r$, and upon the other oblique beds below, and consequently these mouldings were drawn in and contracted very disagreeably, as shown by the dotted line $n q, p q$ being
 taken equal to $m n$. This is a curious proof of the rough neglect of minute circumstances by the Gothic masons of which plenty of other examples might be adduced, and it must be confessed that the false lines so produced are not perceptible from below. I believe we might in many cases reduce very considerably the expenses of our constructions if we had courage to imitate our ancestors in this respect. In the Canterbury example the angle $m p q=110^{\circ}$, and the depth $m n$ of the mouldings is five inches, consequently it may be easily calculated that $p q$ is half an inch too short; and the effect of drawing the point $q$ half an inch out of its true place compared with ten inches, which is the thickness st of the stone, is very perceptible when the stone is looked at upon the ground, but when in its place aloft I have no doubt this error was perfectly unappreciable.

## Section II.-On the Curvature of the Ribs.

The next set of examples which show the necessity of a geometrical system will be found in the construction of the intermediate ribs of vaults, and in the management of the curvature of the ribs generally. The limits of this paper will not allow me to enter fully into the description of the different classes of vaults in the decorative sense, neither is it necessary for my present purpose; I shall therefore briefly state the different steps by which they appear to have been led on from the simple cross-ribbed vault to the fan tracery. The plain cross vault, Roman in arrangement, but pointed and with the addition of ribs upon the groins, is to be found at Salisbury, Gloucester nave, Canterbury choir, Wells nave, Beverley, Westminster choir, and in all the French cathedrals. Simple intermediate ribs were first added between the wall ribs and diagonal ribs, and between the transverse ribs and diagonal ribs. Thus, in fig. 9, a vault with intermediate ribs is represented in a
diagram upon the principle of delineation, which was first employed by Mr. Ware, in his admirable treatise on this subject ${ }^{7}$.

Abкц are the points whence the ribs spring, and between which is given the plan of the vault in a kind of diagonal perspective. AC, вс, KC, LC are the diagonal ribs or great cross springers (termed croisée d'ogives by De l'Orme). a a a $f$ $\mathrm{A} g$ are the intermediate ribs of that spandrel of the vault which lies nearest to the eye in the diagram. These intermediate ribs are termed the tiercerons by De l'Orme, which being a very con-
 venient word, I shall employ. Now in clerestory vaults, the transverse dimension of every compartment is commonly about double that of its longitudinal dimension. AD will, therefore, be the transverse rib of the vault, and AE the rib which lies next the wall of the clerestory or the wall rib, as I have ventured to call it, the formeret of De l'Orme.

In this figure I have shown one tierceron between the transverse and diagonal ribs, and two between the wall rib and diagonal rib; but the number varies in different examples. The figure agrees with the vaults of the choir of Lichfield and the south transept of Hereford.

One tierceron in each space dca and cas is to be found in Lichfield nave and Lady Chapel, Norwich cloister, Exeter side aisles, Lincoln nave, Westminster nave and cloisters, and in the vault at the intersection of the nave and transept at Amiens cathedral. Sometimes three and one are employed, as in Exeter nave, or three and two, as in Norwich nave. Much of the effect and character of these vaults depends upon the curvature of these tiercerons, and also upon that of the diagonal and transverse ribs between which they are placed; and even in the simpler vaults, which have only the diagonal and transverse ribs, this curvature governs the character of the vault by determining the form of the spandrel solid.

It is evident, indeed, that if a given parallelogram is to be vaulted with a groined and ribbed vault, and the crowns of the arches are all to be nearly at the same height, that some geometrical difficulties will be introduced in the management of the forms of arches of such different spans as the transverse, longitudinal, and diagonal ribs, and this difficulty was much greater before the pointed arch was introduced.

[^4]The vault of the castle at Newcastle, already cited and explained, is a parallelogram of twenty-seven feet by twenty, and the three spans are here accommodated by employing semicircular arches and stilting them at their imposts to bring their crowns to the same level. This, as I have shown elsewhere ${ }^{8}$, was the Roman expedient, and was employed in the baths of Diocletian and Caracalla, of course without the diagonal rib. The side aisles of the chapel in the White Tower of London, however, have plain groined vaults without ribs upon a parallelogram, exactly upon the same principle as that of Diocletian's baths, and with the waving groin, which I have, in the passage just referred to, shown to be a necessary result of such an arrangement.

The side aisles of the nave of Peterborough cathedral are examples of the same difficulty, which is overcome without the use of pointed arches by the new expedient of employing a segment less than a semicircle for the diagonal rib, or, in other words, by placing the centre of the circle below instead of above the level of the impost or springing. The parallelogram to be vaulted in this case is about fifteen feet by eighteen feet, but from the massive construction of the piers the bounding ribs are contracted in their spans, especially that on the long side which is next to the nave. The spans of the transverse and longitudinal ribs of the vault are twelve and fifteen feet, and of the diagonal rib twenty-one, and their crowns are nearly at the same level. To meet this difference of proportion between the spans and height, the longitudinal rib is slightly stilted, and the transverse rib very much so; but the diagonal rib is a small segment of a circle, and therefore springs off the abacus at a considerable angle, and in a manner totally at variance with the two neighbouring ribs, which rise from their imposts with a slight inclination backwards, forming horse-shoe arches.

After pointed arches were introduced, the difficulty of adjusting these three arches was greatly diminished; but notwithstanding the possibility of making pointed arches of any proportion of height and span with their centres of curvature upon the impost line, it will be found that the old expedient of placing the centres of curvature above or below the impost line, for the better adjustment of these arches, was still retained in pointed architecture until the four-centred arch was brought in. I shall proceed to examine this more at length, since it involves so much of the characteristic appearance of these vaults.

The ribs in early specimens consist each of an arc of a single circle, but

[^5]in later examples of two arcs of different radii conjoined so as to form the half of what are termed four-centred arches, since the term rib is applied to half the arch. Some ribs are, even in complex vaults, formed of three arcs, as I shall presently show.

In the first place, however, I will speak of ribs of a single arc only.
Ribbed vaults may have horizontal ridges, or they may be domical; that is, the point E of the ridge ( fg .9 ) may be at the same level or may be lower than c, and it may be also curved from c to e, in which case, as far as I know, it is always either straight or concave on its lower side, but not convex ${ }^{9}$. Sometimes, to suit particular cases, as, for example, to admit of a high window, the apex E of the wall rib is thrown higher than c .

Domical vaults are múch more common on the continent than in England, and especially run into excess in the middle age vaults of Italy; however, we have some specimens of them, as, for example, in the nave of Worcester.

The form of the ridge is, however, one of the first things that should be observed in examining an existing vault, and is also one of the first things that must be settled in a proposed vault; for the form of the ridge decides the relative altitudes of the summits or crowns of the ribs.

Now supposing a rib to consist of a single arc of a circle, we may either place the centre of this arc upon the impost level, or we may allow it to be placed above or below that line. But the plan of the vault gives the span of each rib; also, when the height of the vault and the form of the ridges are determined, the altitude of the crown of each rib is also given. If, therefore, the centres are to be upon the impost line, the radius of each rib is given by these conditions; but if the centre may be above or below, we may for each rib take any radius we please.

For let a в ( fig. 9.a) be the span of a rib which is given by the plan, and в с the height of its crown, which is also given, as already explained. Then since the rib is an arc of a circle, which must pass through the two points a and c, its centre must be on the line ed, which is a perpendicular upon the middle point of the chord Ac. If, therefore, the centre of the arc
 is to be on the impost line $A b$, it can only be at $D$, where the two lines ed ab intersect; but if we are allowed to place it above or below this impost line, it may be at any points $d$ or $e$ upon the line ed.

One set of workmen appear to have confined themselves to the first prac-

[^6]tice, and another set to have allowed themselves the second; and it would be a very desirable thing to ascertain the exact curvatures of the ribs of a great many of these vaults, at the same time noting also their general character and appearance, with a view to determine the practice of the different schools, as well as to study the results with a view to improve modern practice, which is perhaps more deficient in this matter of curvature than in any other.

In the above proposition I have supposed that the form of the ridge was settled, or at least the altitudes of the crowns of the ribs, before the radii were determined, but we may suppose various other methods of proceeding; for example, that the pointed arches of the ribs should all be similar, that is, that they should all have the same ratio between their span and altitude. This rule produces a highly domical vault, because the crown of the diagonal rib is necessarily thrown very high above that of the other two. The Italian mediæval vaults appear to be governed by this rule as nearly as the eye can detect. But in England the ridges of the vaults are most commonly level.

In some specimens, however, other principles govern the relation of the curvatures ; for example, in plate 77, Pugin's Specimens, is a diagram to show the curvature of a vault on the east side of the cloisters at Westminster. In this vault the ribs are each a single arc of a circle, with the centre upon the impost line, and the diagonal rib has the same radius as the transverse ribss ${ }^{10}$, so that the vault is highly domical in its structure; it has no ridge ribs, but the ridge necessarily rises to the centre of the vault, since the crown of the diagonal rib is by this construction thrown so much higher than that of the transverse ribs. This principle of employing a common radius for the diagonal and transverse ribs agrees with many other examples, as I shall presently show.

On the other hand, there is also in Pugin's Specimens (vol.ii. p. 29) an excellent detailed drawing of the vault of the Lady Chapel, Southwark, in which the ridges are horizontal. The compartment is twenty feet six inches square, and the spans of the bounding arches are fourteen feet six, and of the diagonal arch twenty feet eight inches. The ribs or semi-arches are each formed of a single arc of a circle struck from a point eight inches below the impost level, and with a radius of nine feet five inches. The entire diagonal arch is struck from a single centre two feet seven inches below the impost level with a radius of

[^7]eleven feet three inches, and is therefore a segment of a circle instead of being a pointed arch.

In many examples I have seen that the transverse rib is struck from centres above the impost level, and therefore its arch is a horseshoe, and in the same vault the diagonal rib will often have its centres below the impost level. On the contrary, in the north transept of Hereford cathedral the centres of all the vault ribs are very considerably below the impost level ${ }^{11}$. I have shown that when the centres are allowed to be placed out of the impost level, there is a choice of many radii for the curve of a rib of given span and altitude, and of course as the radius increases, the rib approaches nearer to a straight line. But the effect upon the general form of the spandrel solid fobe fig. 3, or doea, fig. 9, is the principal point to be attended to, and this is best appreciated by considering the form which its middle plan assumes, that is, the form of a horizontal section taken at about half way up the arches, as at $\mathrm{K}_{\mathrm{Q}}$, fig. 3, or perhaps a little higher, as at $m n p$, fig. 9. If the ridge ribs are level, as in this figure, the plan at the crown of the vault DCE will necessarily be a rectangle, but by different arrangements of the curvature of the ribs we may make the middle plan $m n p$ assume any figure we please.

Thus in the simple vault with transverse and diagonal ribs only, as in fig. 3, we may, by making ab more or less curved, cause it either to lie between $A D$ and $A E$ in such a manner that a horizontal rod which touches these two ribs half-way up, will also touch $A B$; or we may make the middle point of $A B$ lie in front of such a rod, the effect of which is to make the spandrel solid appear convex in front, and approach to the form of a fan-vault, which is the case in Pugin's second example above quoted; or else we may make the middle point of $A_{b}$ lie in the opposite direction, so as to make the solid concave in front: of each of which dispositions examples may be found. Similarly, when intermediate ribs are added, as in fig. 9, they may be disposed in various ways so as to affect in a similar manner the form of the middle plan, and throw it into various figures. By geing to the upper surface of a vault, and looking down into the pockets or cavities which lie over the shafts, and which are in fact the insides of the spandrel solids, the middle plan will be

[^8]pore distinctly seen than from below, and in making architectural notes its general form at least should always be recorded.

Since this middle plan affects the character and appearance of the vault so paterially, I will show how, by a very simple construction, it may be employed in giving any desired form to the spandrel solid; and although I do not mean to say that it was so employed by the middle age architects, it may be made useful in examining and comparing their works.

Let fig. 9 (page 13) be a vault of which the plan is given, and of which the beight of the apices of each rib, $\mathrm{E}, g, f, \mathrm{c}, e, \mathrm{D}$, are also determined, as well as the middle plan $p, v, s, n, r, m$. The ribs being each a single arc of a circle, it is required to find the radius and centre of each rib. Now we have given for each rib three points through which it must pass: namely, the springing at a, the middle point, and the apex; therefore the question reduces itself to the common workman's problem, Given three points to describe an arc of a circle through them ${ }^{12}$. The complete construction for a vault with tierccrons is shown in fig. 10.

Let $a b c d$ be the plan


FIG. 10. of the vault, af the diagonal rib, afag the tiercerons, an the clerestory rib. Draw lm of the required figure for the transverse rib LK , and also $\mathrm{ZPQ}_{\mathrm{P}}$ the figure of the clerestory rib, which (as is shown at page 7) will be raised upon stilts, as at a p. Find the point $s$ at half the height км of the transverse rib, and draw r $s$ perpendicular to a $k$. Draw the middle plan R $w z$

[^9]of the spandrel according to the required form, also Lz the plan of the ribs upon the abacus.

To find, then, the curvature of any rib, as for example of the tierceron af. If the ridge riber be horizontal, the height of the point F will equal that of the point k. Set off $\mathrm{f} f$ perpendicular to A F and equal to km ; and because $r$ is the seat of the perpendicular from the point where the tierceron passes through the middle plan, set up $r t$ perpendicular to AF and equal to $\mathrm{R} s$. Then will $\mathrm{L}, t$ and $f$ be three points through which the required rib passes, and the centre $h$ found by the common rule, will be the centre of curvature of the rib. In the same manner will be obtained the centre $g$ of the diagonal rib a me. The arrangement of the rib upon ag is more difficult, for as it lies between two ribs, $A_{H}$ and $A E$, one of which is stilted above the other by a distance zP , it is necessary to stilt also the rib upon $\mathrm{A} G$ by about half that distance. Set up, therefore, a perpendicular $т k$ equal to about half zp, and obtain the other two points $b$ and $d$ as before. $z$ will be the centre of curvature of $k b d$.

Any other forms may be given to the middle plan, as shown at the angle c of the same figure by $\mathrm{x} y z$, or at xwz , or at s and L in fig. 9. I have drawn the diagram fig. 10 upon the hypothesis that the middle plan shall be taken exactly at half the vertical height of the arch, and that the ridges shall be level. If, however, the ridges be curved into any given figure, that curvature will give the separate altitudes of the crowns of each rib, and therefore, instead of setting up $l \boldsymbol{m} f f$ e $e$ G $d$ all equal, they must be made respectively equal to the altitudes thus obtained from the given ridges.

Also it may be better to take the middle plan higher up, because the form of the spandrel will be developed more clearly; for example, let it be taken at a point midway between the springing and the crown of the arch measured upon the arch itself. The point $s$ being assumed half way between L and m, the remainder of the process will be the same, but the middle plan $\mathrm{R} w z$ will come out larger in proportion to the rest of the figure. Again, I have drawn the transverse rib first, but it may be better to begin by drawing the diagonal rib $\mathrm{A} e$ first, as that, from its greater span, is the most difficult to manage ; in which case the point $m$ must be assumed at the proper height for the middle plan, which will then be derived from the perpendicular $m w$ instead of from the point $s$ and perpendicular $s$ r. I have reason to believe that the diagonal rib was often the first rib settled by the mediæval architects, as it so often appears in the form of a segmental or semicircular arch. The want of its point is concealed by the boss.

A common method of executing modern Gothic vaulting, especially in plaster, has been to obtain the form of the diagonal ribs, as ac, fig. 9, by ordinates from those of the transverse ribs ad, aE. The effect of this is, that the entire vault appears as if it consisted of two waggon vaults, crossing at right angles, and a horizontal rod moved from top to bottom in contact with any pair of opposite arches вм AE will touch the diagonal ribs bс AC and the connecting vaulting surfaces throughout. This is the genuine principle of the Roman and Italian groined vault, but is altogether foreign to the principles of Gothic architecture, in which every rib should spring as a separate and independent arch, and in which the elliptic curves produced by this method are totally at variance with the characteristic forms of the style.

De l'Orme first taught this method of projecting diagonal groins by ordinates, and applied it to the wooden cradling of an Italian vault in his Nouvelles Inventions pour bien bastir (1578, p. 20). This form of vault, however, is a kind of square dome (cul de four quarrée), but requires the same diagonal curves for its groins (or rather ridges) as the Roman cross vault. The latter vault is not amongst the examples of stone-cutting in De l'Orme's book. The square dome is also given by Maturin Jousse with the same method of ordinates (Le Secret d'Architecture, 1642, p. 127), who also gives, for the first time, the Roman groined vault (Voûte d'Areste quarrée, p. 157) under different shapes and plans, which becomes a standard example in all the succeeding writers upon the subject, as Derand, De la Rue, Frezier, \&c. But this method was never intended by them to be applied to Gothic rib vanling. The Gothic style had grown out of use when De l'Orme wrote; but as a matter of curiosity he has described the carves of a Gothic vault (in ch. viii.), in which every rib is perfectly independent of the other in its curvature. Each rib consists of a single arc of a circle whose centre is upon the impost level, and they cannot be therefore connected by projections. They all form pointed arches of differeut proportions, with the exception of the diagonal arch, which is very nearly a semicircle. The vault in question consists of diagonal ribs, with one intermediate tierceron in each angle, as in fig. 10, but that the plan is a complete square. The ridge-rib or lierne ex is not horizontal, but slightly curved, so that the height of k above the impost plane is to that of E as about nine to eleven. The curve of the diagonal rib was first drawn (as De lorme bimself says), then apparently the relative altitudes of the points $E$ and $F$ (that is, the lines ee ff ), were determined, by drawing the proposed curve of the lierne; and as the centres of the arcs are upon the level of the imposts AEAF, nothing remained but to draw the arcs of circles each through the two
points $\mathrm{T} e \mathrm{~L} f$. For, as $\mathbf{I}$ have already remarked, only two points are required to determine the circles when coupled with the condition that the centre shall be on the line of the impost.

This may have been the genuine French Gothic method, but in our English examples the centres are commonly placed without respect to the impost level, and the general forms of the vault are different from those which are produced in this manner, as already mentioned. It is easy to see, however, that this method of De l'Orme is very far removed from the projection system, and that from its simplicity it was in all probability, one of the rules of the olden time.

Derand has also a chapter upon the Gothic vaults, but it appears to have been a mere comment upon De l'Orme, and indeed he wrote too late (1643) to be able to collect any genuine traditions of the Gothic methods; whereas De l'Orme himself lived so close to the Gothic period ${ }^{13}$ that he might well in his youth have been taught by Gothic masons. Derand, however, says, that in this style the ribs are always made arcs of circles, elliptical or other curves being inadmissible, although he can see no reason why they should not be used, but that their effect upon the eye is not so agreeable (p. 177). He then proceeds to give the method of describing the arcs, which is manifestly a mere amplification of what De l'Orme had said.

De la Rue is silent upon the subject, but Frezier, 1738, has given the same: matter as Derand, amplified, and with many very sensible remarks appended thereto ${ }^{14}$.

It is not, therefore, from the French writers that the absurd application of De l'Orme's projection method to Gothic ribs is due; I believe this honour must be awarded to the ingenious Mr. William Halfpenny ${ }^{15}$, who, living at a period when Gothic architecture had sunk into complete neglect, may very well be excused for having misapplied as he has done the projection system to the finding the " Mitre Arch of a Regular Groin when the Intersecting Arches are Gothick ones" (p. 16). In this he has been followed by Mr. Nicholson ${ }^{16}$, who has also taken much trouble to construct a Gothic vault with cuniconcidal surfaces and upon other fanciful hypotheses, which, as they produce curves for the ribs totally different from the genuine ones, can answer no purpose but that of destroying the mediæval character of the work ${ }^{17}$.

[^10]The south walk of the cloisters of Westminster has a vault, the ribs of which approach more closely in their appearance to projected ribs of this kind than any I have seen elsewhere, and I therefore took some pains to examine them, and found that they certainly were not so projected, but that the effect ${ }_{\text {Is }}$ due to the form of the middle plan, which is very nearly a square. Neverthe[ess the aspect of this vault is exceedingly flat and vapid compared with the bolder arrangements of the vaults in the other walks of the cloister.

In figs. 10 and 13 I have placed the elevation of each curve upon the plan of its own rib. This neat arrangement is employed by De l'Orme in many of bis diagrams, and has been followed by all writers up to the present time. Of pourse, by adopting it, I do not mean to assert that it was followed by the Middle Age workmen, but rather I have employed it as a compendious and universally understood formula for setting out. The methods given in these figures would be the same if the curves were drawn separately by the side of the plan, which De l'Orme has actually done in his Gothic diagram, after the manner of $f g .11$ or 16 .

It may be observed also that he has taken the pains also to draw moulds for the stones of the vaulting surfaces or pendentifs as he terms them. This, according to his own confession, was very seldom done, these vauits being more usually formed of brick or of stones so small or at least so thin that the curvatures and angles of their beds might be neglected, which indeed is plainly the case in all the existing English examples that I have examined. It must be clearly understood that $\mathbf{I}$ do not suppose the method of fig. 10 to be a restoration of an ancient practice. My object is to show clearly the mutual geometrical dependence upon each other of these three things which belong to a vault upon a given plan with single-arc ribs, namely, the form of the ridges, the middle plan of the spandrel, and the curvature of the ribs. Of these, if any two be determined, the other follows as a matter of course. It seems, therefore, most natural for us to select the two for determination which are the most easily appreciable by the eye, and these are the ridges and the middle plan. The method which I have given proceeds in this manner, and furnishes Q general process by which we may imitate the various arrangements of these two things in the specimens of different ages.

In the old time, one style alone was practised in each period, and a few simple rules were sufficient for the purpose. The change or improvement of One or more of these rules introduces new features and new characters, but Which still are alone employed as long as they last, and until they are in turn superseded. But we, imitators of all styles, must have more comprehensive
and flexible rules, capable of imparting to our works the characters of every age in turn. This necessarily gives to the methods which we invent and employ a much greater degree of complication than is likely to have belonged to the rude practical geometry of the Middle Age workmen; each of our con* structions being in fact a general formula, which includes many particular instances, every one applicable to a separate period.

Notwithstanding, therefore, that I have employed the middle plan in determining the curvatures, I think it most likely that the different forms of the middle plans which may now be observed resulted from different rules for finding the centres and radii of the ribs, which were employed by the different schools of workmen according to their age or nation. In fact, in some examples the geometrical difficulties are very clumsily botched over, while in others the ribs are so beautifully adjusted in their relations that the greatest possible boldness and grandeur of effect is brought out, and to a degree which never appears after the four-centred arches are introduced.

When the centres of single-arc ribs are placed upon the impost level, we have no longer the liberty of disposing at pleasure both of the form of the ridge and of the middle plan of the spandrel. One of these being settled, the other follows as a matter of course. It would lead me too far into geometrical investigations for the nature of the present paper were I to follow out these principles to their consequences. This I must leave for a future opportunity. But in this section my purpose is rather to direct investigation towards certain points and characteristic features in the adjustment of the ribs, which appear to me to exercise a great influence over the form of the vault, and even over its æsthetic character, and to the neglect of which may be attributed much of the feebleness of appearance in modern Gothic vaulting.

In vaults of the Roman and Italian styles, of which the groins are without ribs, the vaulting surface is the leading feature, and the disposition of it the only object to be attended to. But in Gothic ribbed vaults, on the contrary, the ribs are the principal features, and the surface of the vaults subordinate: To maintain this subordination of the vaulting surface to the ribs, the latter should branch off from the abacus with the greatest possible appearance of mutual independence as separate arches, an appearance which is better given by single-arc ribs than by double-arc ribs or semi-fourcentred arches. Also the vaulting surfaces or pannels of contiguous compartments should by no means have the appearance of continuity, which is given by the projection system, but which immediately suggests the idea that the surfaces really cons stitute the mechanical vault independently of the ribs, which seem to have
been subsequently added, and might be removed without destroying the vault; instead of which the ribs really support the vault, and should appear to do so in the decorative as well as in the mechanical construction.

The apparent mutual independence of the ribs is increased in the best specimens by the manner in which they start from the abacus, some being more prominent than others. This is easily managed in the diagram fig. 10, by placing the feet of the curves at different distances from the centre a of the abacus.

I have shown (to return from this digression) that a very simple geometrical process enables us to obtain the curvature and form of every rib from the two plans and the heights of the ribs combined, but that without some such process they could not have been arranged with so manifest a power over the effect of the combination as the existing examples make evident.

In vaults, however, whose ribs consist each of a single arc of the circle, every rib of the group will spring off from the common abacus at a different angle, if the centres of their respective circles be some above and some below the level of that abacus. In the first case the ribs will be slightly horseshoed or stilted, and in the second will start abruptly forward. These discrepancies may be seen in the majority of these vaults, and it was perhaps to remedy them that four-centred arches were introduced for the ribs, so that each rib shoald consist of two arcs of circles combined so as to have a common tangent at their juaction. For when ribs are thus formed of two arcs, any number of them may start from the abacus at the same angle and even with the same curvature, and yet may have each a different height or span under certain simple limitations.

Now a four-centred arch, or, which is the same thing, a two-centred rib, may have its upper radius adjusted to accommodate different spans and heights in two ways, when its lower radius remains constant.

Frrst, fig. 11, let ADCB be the quarter plan of a vault, and, for simplicity, suppose the ridge rib дв level, so that all the ribs will have the same height at the apex. Transferring the lengths of the ribs upon the plan to $\mathrm{A} e$ and $\mathrm{a} b$, set up oheg bf equal to each other and to the height of the ribs, and let the arc A E, whose centre is $e$, be the given lower circle of the ribs, which is to be the same in all. Produce e $e$ to H , then any circle struck through E whose centre is upon this line will touch the circle AE at e, and thus answer the purpose of the upper circle of the rib. By taking
different radii for this upper circle, therefore, we can make the ribs pass through the points $h g f$, as shown in the diagram, where $\mathrm{H}, \mathrm{G}$ and F are the respective centres of the ribs $\mathrm{E} h \mathrm{E} g \mathrm{E} f$.
${ }^{18}$ In this example the ribs not only start all with the same curvature, but the change of curvature is made at the same height e in all, consequently the upper circles have all a different radius.

Secondly, let it be required that the upper circles shall all have the same radius as well as the lower circles. In fig. $12, \mathrm{AD} \mathrm{D} h$ is the span and height of one rib, A в $\mathrm{B} f$ those of another rib. c is the centre of the lower circle $A b$, which is common to both ribs. But as the upper circles are to be also the same, the curvature cannot change at the same height in both. $b_{\mathrm{H}}$ being the given radius, the centre of one upper curve $b h$ is at a , and of the other $c f$ at F . It is easy to see that these centres will lie in the circumference of a
 circle whose centre c coincides with that of the lower circles and whose radius CH is the difference between the radii of the lower and upper circle of the ribs; also a centre can be found at once, so that its arc shall pass through any given point, as $f$, by intersecting the locus $\mathrm{H} F$ with an arc whose centre is $f$ and radius that of the upper circles.

Which of these principles of adjustment were employed by the Middle Age architects I do not know, but it would be very desirable to measure accurately a good number of examples, to ascertain whether the curvature of the upper circles is often the same in all the ribs as well as the lower.

There are two working drawings of vaults, with intermediate four-centred ribs, in Pugin's Examples, upon which the curvatures of the ribs are indicated, apparently with great accuracy. In one of them, the gateway of Magdalen College, the ridge ribs are horizontal, and consequently the arched ribs all of the same height. Each rib consists of two arcs of circles, but the lower circle is of very small diameter with respect to the upper circle, its radius being about oneninth of the latter, and the same in all the ribs. All the upper circles of these ribs appear in this drawing to have the same radius, which is equal to the span of the transverse arch.

The other example is the groining of a bay window of Eltham Palace, also having one intermediate rib between each diagonal and wall rib. The ridge

[^11]ribs are not horizontal, consequently in describing the curves of the arched ribs, the ridge ribs were probably first determined to give the heights of these ribs. Each rib, being half a four-centred arch, consists of two arcs of circles, the radius of the lower arc being a little less than half that of the upper arc, and this latter radius is again equal to the span of the smallest arch. These may be accidental proportions; however, the two radii are respectively the same in every rib, and consequently the different heights and spans are accommodated, as in fig. 12, by employing different proportional lengths of the two circles in each rib. The centre of the lower circle is in all placed on the impost level, and this I believe to be universal in four-centred arches.

The wall rib, in examples of this class, is often different both in its lower circle and upper circle from the other ribs, and in some cases the accommodation of heights is but clumsily effected. In the vault of Queen's College gateway at Cambridge, fig. 16, the ridge ribs are horizontal. The elevations of the principal ribs are shown upon $\mathrm{a}_{\mathrm{A}}$, where $\mathrm{G} m$ is the curve of the wall rib af, $G n$ that of the transverse rib Ga, and $a q p$ that of the diagonal rib $G$ к. The wall rib has a lower circle whose centre is $t$, and radius $t k$ greater than those of the other ribs, and its upper circle $k m$ is different, so that this rib is quite different from the others. The diagonal rib, transverse rib ga, and tiercerons have all the same lower circle $\propto y$ with radius $s y$, and the same upper circle $y n$, so that the curvatures of these ribs are identical up to $q$, but the different spans are accommodated by breaking the curve off abruptly, as from $q$ to $p$. As this change of direction takes place near the bosses


FIG. 16. $\mathrm{B}, \mathrm{C}, \mathrm{D}$, it is scarcely perceptible unless closely looked after.

The vaults of the side chapels of King's College, Cambridge, are excellent specimens; those towards the west end, namely, four on the north side and three on the south side, have four fan-vaults similar to that of the chapel itself, and the two extreme north-eastern chapels have very elegant vaults of an earlier date than the great vault. The remainder of these small chapels are Vaulted with plain vaults, which are included in the same contract with the fan vaults, and are specified as to be made of " a more coarse werke" (vide contract in Britton's Ant. vol. i.) These chapels are parallelograms of twenty feet six inches by twelve feet. The plain vaults have intermediate ribs or tiercerons, precisely in plan the same as fig. 9. The ribs are, however, two-
centred and all of the same curvature, and the ridge ribs consequently have the figure of fig .20 . The vault is built of rib and pannel work. The vaults of the two north-eastern chapels are of the class which I have in a succeeding part of this paper denominated lierne vaults, and have an elegant pattern of the stellar kind, which is so contrived as to form not only stars round the angles, but also a six-rayed star round the centre. It has in all twenty-three bosses. This vault is also built of rib and pannel work, and the boss stones are formed on the principle of fig. 16, each stone including a portion of pannel between the stump of the ribs. The curvatures are beautifully managed. The ridges are both horizontal, and the ribs all leave the impost with the same curvature, and apparently their upper or crown curves have all the same radius, the different spans being accommodated on the principle of fig. 12. The transverse arch is very nearly equal in height to its own span. On the whole, these vaults are excellent specimens, but are seldom seen, as the chapels are shut up and neglected, and used as a workshop or lumber place.

I am indebted to the kindness of Charles Barry, Esq. for the exact measures and curvatures of the vault of the crypt of St. Stephen's Chapel, at Westminster, which follow. The details and arrangements of this crypt have been published in various well-known works, and therefore the diagram plan of one quarter of a bay, in fig. A, will be sufficient for our present purpose, which is merely to give the curvatures of the ribs.

A is the central boss of the vault. BCDE, the other bosses.
$F$, one of the piers.
The respective diameters of the bosses are as follows :-

Boss a, 3 feet 3 inches diameter.

| B, 1 | 5 |
| :--- | :---: |
| C, | 1 |
| D, | $\mathbf{1}$ |
| E, | 7 |



The position of c on the plan is given by the perpendiculars $\mathrm{c} m \mathrm{c} n$, from the centre of the boss upon the lines an $\mathrm{D} d$ respectively, of which $\mathrm{c} m=3 \mathrm{ft}$. $6 \frac{1}{2}$ in. and $\mathrm{c} n=3 \mathrm{ft} .11 \mathrm{in}$. The ribs $\mathrm{AD}_{\mathrm{D}} \mathrm{A}$ в and BE are all straight, but DC and Ac are slightly curved; the first having a rise of 1 in . and the second of $\frac{7}{8}$ in. The curvature of the ribs а $a$ в $b$ с $c d$ е $e$ are shown in the following. table, the explanation of which will be found on the succeeding page.

Curvatures of the Ribs in the Crypt of St. Stephen's Chapel, in feet and inches.

|  |  | Rib a ${ }^{\text {a }}$ | Rib ${ }^{\text {d }}$ | Rib co | Rib в $b$ | Rib ee |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| в 6 | A B | $\begin{array}{cc} 1 & 8 \\ 10 & 8 \frac{1}{2} \end{array}$ | 10 $\begin{array}{r} \\ 10 \\ \hline\end{array}$ | $1 \begin{array}{ll}10 & 8 \\ 4 & 45\end{array}$ | 11 17 | $\begin{array}{ll} & 3 \\ 11 & 1\end{array}$ |
| c $c$ | в С | 1 17 <br> 10 $7 \frac{1}{8}$ | $\begin{array}{ll} 1 \\ 10 & 3 \end{array}$ | 1 61 <br> 9 $9 \frac{1}{8}$ <br>   | $\begin{array}{rr} 1 & 5 \frac{1}{2} \\ 10 & 6 \end{array}$ | $\begin{array}{cc} 1 & \\ 10 & 6 \frac{1}{8} \end{array}$ |
| D $d$ | C D | $\begin{array}{rr} 1 \\ 10 & 4 \frac{3}{4} \end{array}$ | $\begin{aligned} & 1 \\ & 9 \\ & 11 \frac{1}{4} \end{aligned}$ | $9{ }_{3 \frac{7}{8}}^{11}$ | $\begin{array}{lll} 1 & \\ 9 & 11 \frac{1}{4} \end{array}$ | $\begin{aligned} & 1 \\ & 9 \\ & \\ & 11 \frac{3}{4} \end{aligned}$ |
| E $e$ | DE | $\begin{array}{rr} 1 & \\ 10 & 15 \end{array}$ | $\begin{array}{ll} 1 \\ 9 & 6 \frac{1}{4} \end{array}$ | $\begin{array}{ll} 1 & 1 \\ 8 & 7 \frac{7}{8} \end{array}$ | $\begin{array}{ll} 1 & \\ 9 & 3 \end{array}$ | $\begin{array}{ll} 1 & \\ 9 & 15 \end{array}$ |
| $\mathrm{F} f$ | EF | $\begin{array}{ll} 1 \\ 9 & 9 \frac{3}{4} \end{array}$ | $\begin{array}{ll} 1 & \\ 9 & 0 \frac{5}{8} \end{array}$ | $\begin{array}{ll} 1 \\ 7 & \\ & \frac{1}{8} \end{array}$ | $\begin{array}{ll} 1 & \\ 8 & 5 \frac{3}{8} \end{array}$ | $\begin{array}{ll} 1 & \\ 8 & 1 \frac{3}{4} \end{array}$ |
| G $g$ | FG | $\begin{array}{ll} \hline 1 & \\ 9 & 4 \frac{1}{4} \end{array}$ | $\begin{array}{ll} 1 & \\ 8 & 6 \end{array}$ | $\begin{array}{ll} 1 \\ 7 & 0 \frac{3}{8} \end{array}$ | $\begin{array}{ll} 1 & \\ 7 & 6 \frac{1}{4} \end{array}$ | $\begin{array}{ll} 1 & \\ 6 & 11 \end{array}$ |
| H $h$ | G H | $\begin{aligned} & 1 \\ & 8 \\ & 10 \frac{1}{2} \end{aligned}$ | $\begin{array}{ll} 1 \\ 7 & 9 \frac{1}{2} \end{array}$ | $\begin{array}{ll} 1 \\ 6 & 0 \frac{3}{8} \end{array}$ | $\begin{array}{ll} 1 & \\ 6 & 3 \frac{1}{2} \end{array}$ | $\begin{array}{ll} 1 & \\ 5 & 1 \end{array}$ |
| ¢ $i$ | H 1 | $\begin{array}{ll} 1 & \\ 8 & 3 \frac{5}{8} \end{array}$ | $7{ }_{7}^{11}$ | $\begin{array}{lr} 1 \\ 4 & 7 \frac{5}{3} \end{array}$ | $\begin{array}{ll} 1 & \\ 4 & 10 \end{array}$ |  6 <br> 3 $8 \frac{3}{4}$ |
| к $k$ | I K | $\begin{array}{lll} 1 & \\ 7 & 7 \frac{7}{8} \end{array}$ | $\begin{array}{ll} 1 & 1 \\ 6 & 1 \frac{1}{8} \end{array}$ | $\begin{array}{ll} \mathrm{I} & \\ 2 & 8 \frac{1}{8} \end{array}$ | $\begin{array}{ll} 1 & \\ 2 & 8 \frac{1}{4} \end{array}$ | $0^{6}$ |
| $\mathrm{L} l$ | K L | 1  <br> 6 91 <br> 8  | $\begin{aligned} & 1 \\ & 4 \\ & 10 \frac{1}{2} \end{aligned}$ | $0^{9 \frac{7}{8}}$ | $0^{8}$ |  |
| м $m$ | L M | $\begin{array}{ll}1 \\ 5 & 8 \frac{3}{4}\end{array}$ | $\begin{array}{ll} 1 & \\ 3 & 5 \end{array}$ |  |  |  |
| * $n$ | M N | $\begin{array}{ll}1 & \\ 4 & 5\end{array}$ |  6 <br> 4 4 <br> 8  |  |  |  |
| oo. | NO | $\begin{aligned} & 1 \\ & 2 \end{aligned} 10 \frac{1}{2}$ | 1 1 $7 \frac{1}{2}$ |  |  |  |
| р $p$ | $\bigcirc \mathrm{P}$ | 1 $\quad 6$ | $0^{5}$ |  |  |  |
| Q $q$ | PQ | $0^{6}$ |  |  |  |  |

These curvatures were measured by vertical ordinates which rise from a horizontal base at the level of the abacus or impost line of the pier F , and are for the most part taken at a distance of one foot apart, but in some places this equal spacing of the ordinates has been interfered with by local circumstances or obstructions.

The table is arranged in this manner: ma, fig. B, being one of the ribs, let ma be the horizontal line or base, and A $a$ the central ordinate, therefore $a$ will be the crown of the arch or rib. в $b$ с $c \mathrm{D} d$, \&c. are the


FIG.B. other ordinates in sufficient number. Then the feet of these ordinates are indicated by the capital letters in order, beginning with a for the central one; and the upper extremity of each ordinate is indicated by the small letter which corresponds to the capital at its foot. In this way $\mathrm{A} a, \mathrm{~B} b, \mathrm{c} c, \& c$. will indicate the height of the ordinates, and $\overline{\text { a }}$, в с, с d , \&c. their respective horizontal distances. The ordinate at the springing, as $m$, will be necessarily equal to zero. By following this system, a great number of curvatures may be arranged and published without requiring engraved figures.

I have laid down these ordinates upon a large scale, with a view to determine the radii of the circles. However, it will always be found in this operation that the irregularity of the workmanship and the settlement of the work will occasion such deviations from the original form, as to leave the exact radius and centre of the arc a matter of some doubt. For this reason, the fairest and best plan is to publish the ordinates in every case, without attempting to form hypotheses; for it will only be by collecting a great nümber of such examples, and comparing them, that we can hope to deduce general rules. I will merely state the radii as far as I have been able to determine them.

The ribs $\mathrm{A} a \mathrm{D} d$ (fig. A) are semi-fourcentered arches, but the ribs с $c$ в $b$ $\mathrm{E} e$ are each of them arcs of one circle. The ribs $\mathrm{A} a \mathrm{D} d$ agree so nearly in form, that it is evident that their curvatures are intended to be the same as far as they go together, for $\mathrm{A} a$ is of course of a larger span. The centre of the upper curve of a $a$ is immediately below its crown, so that in fact this diagonal rib is a semi-three-centred arch. This agrees with the example already quoted from Pugin of St. Saviour's vault, and is, I believe, a pretty general practice. The radius of this upper curve is 15 ft . 10 in ., and its centre is 5 ft . below the impost level. The radius of the lower circle is 10 ft .8 in ., and the line which contains the two centres makes an angle of 38 deg. 20 min . with the base or

impost level. The radius of $\mathrm{E} e$ is 11 ft .11 in . and its centre is $3 \frac{\mathrm{I}}{2} \mathrm{in}$. above the impost line. The radius of в $b$ is 14 ft . 9 in . and its centre is 2 ft .7 in . below the impost line. The radius of c $c$ is 13 ft .6 in . and its centre is 2 ft .6 in . below the impost line.

The height of the central ordinates, that is, of the crowns of the arches, cannot be measured, because the bosses cover them, consequently they are omitted in the table.

## Section III.-On the Ridge Ribs, Liernes, and Bosses.

The intermediate ribs or tiercerons which have been considered in the last section are each of them semi-arches springing from the same abacus as the principal ribs, and, as far as their separate voussoirs are concerned, no new difficulty is introduced by them into stone-cutting.

But, in addition to them, we find other intermediate pieces or short ribs introduced in the vault, by which an enriched appearance is given to it ; and which, instead of springing from an abacus, merely connect the other ribs at different points of their height. One set of these pieces, the ridge ribs, occupy the ridges of the vault, and therefore connect the crowns of the ribs alone. Another set, which I shall term the Liernes, connect the ribs at other points, or may connect the crown of one rib with some intermediate point between the crown and springing of another rib. For example, in fig. 16 (p.27), the gateway of Queen's College, Cambridge, k m , k N , are ridge ribs, and $\mathrm{AB}, \mathrm{bc}, \mathrm{c} \overline{\mathrm{j}}$, $\mathrm{de}, \mathrm{EF}, \mathrm{E} \mathrm{H}$, are liernes.

Now as these pieces are not necessarily either straight or horizontal, their lengths and the angles of their beds cannot be obtained immediately from the plan, but necessarily require the use of a geometrical projection combined from the plans and elevations of the ribs.

The term Lierne is applied by De l'Orme to the short-ridge ribs which form a cross at the summit of the vault which he has given as an example, and which correspond to ED and DF in fig. 13 below; but I am inclined to think that the same word would have been used for the other short ribs, such as em, mF, if they had occurred in his drawing. The word is a carpenter's word, and is also applied by De l'Orme, in his "Inventions pour bien bastir," to the short pieces which at different levels tie the wooden arches of his roof together, " les Liernes qui lient et tiennent en raison les dictz aiz pour faire les hemi-
cycles." ${ }^{16}$ They are thus apparently liens, short ties (rather struts), or connecting pieces; and the word, whatever may have been its application, is a very convenient term for those ribs of a vault which do not spring from an impost, in opposition to those which do, and which are, as we have seen, termed transverse ribs ad (fig. 10), wall ribs or formerets a B, diagonal ribs or croisée d'ogives Ac , and tiercerons or intermediate ribs, or in general branches or nervures. So many of the terms that have been recovered of the Middle Age workmen in England are Norman French, that I see no objection to the employment of those French words which we cannot doubt to be genuine; I shall, therefore, venture to call these short ribs liernes, applying the term to every rib that does not spring from an abacus, excluding from it, however, the ridge ribs.

The intersections of the tiercerons with the ridge ribs and with the liernes are commonly covered by bosses. Every boss stone being thus a centre whence portions of ribs diverge at different angles and in different vertical planes, is in itself a complex figure, which likewise requires the employment of a geometrical system. It may be observed that the use of bosses is much

older than the introduction of the liernes or even of the ridge ribs into the vaults; but as the geometrical process for obtaining the forms of all is very
nearly the same, I will show what I conceive to have been this process, and then explain the evidence by which one part of it at least is proved to have been employed by the workmen of the period in question.

Let $\mathrm{ABCD}, f \mathrm{fig} .13$, be the plan of one quarter of a severy, of which the ribs spring from A. AD is the diagonal rib, and AE AE are intermediate ribs or tiercerons, springing also from A. EM FM are liernes, and it is required to obtain the shape of the boss stone at the intersection m , and also the forms of the two liernes fmem, the curvatures of the ribs having been previously determined. Upon D set up da perpendicular to AD and equal to the height of the centre of the vault; and upon ad describe ag, the elevation of the curve proposed for the diagonal rib. Since the middle planes ${ }^{17}$ both of the ribs and of the liernes are vertical, their intersection will be a vertical line, the seat of which upon the plan will be m. Call this line the axis of the boss, and draw mK perpendicular to $\boldsymbol{H D}$ to represent this axis in the middle plane of the diagonal rib.

The carving which decorates a boss is commonly included in a circular or rather spherical form, but the centre of this circle is not always upon the axis of the boss; on the contrary, it is placed so as best to cover the meeting of the ribs. The circle described at m represents the space intended to be occupied by the carving. Mark points $a b c a$ upon the rib and liernes to indicate the place of the joints on the lower surface of the boss stone, and project the points $a b$ upwards to meet the curve of the rib at $e$ and $f$. From the centre of curvature L of the rib, draw the joint lines $\mathrm{eg} f \mathrm{fh}$, and draw hg perpendicular to the axis of the boss for the upper surface of the stone, which, as will presently appear, is always horizontal. This surface line must be drawn at a sufficient height above $k$ to leave the proper extent of bed at the small end of the stone eg. Project the points $g$, $h$, downwards to meet the plan Ad of the rib at $k$ and $l$. Then will $k l$ be the length of the upper surface of the stone in its proper relative position to the axis of the boss m .

This double projection, for example of the point $a$ upwards to $e$, and of $g$ downwards to $k$, is of constant use in this method, for the place of the joints is in every vault determined by the nature of the pattern which decorates its lower or visible surface, as we shall see more clearly in the complex fan vaults. But the shape of the stone is obtained from the plan or mould of its upper surface. To obtain this mould, therefore, we must begin by placing the joints upon the decorative surface, and then, by this system of upwards and down-

[^12]wards projection, we can obtain the intersection of the joints with the upper surface of the stone, and thence its upper plan, as we have begun to do in the example before us, where the joints $a$ and $b$ were first placed so as to leave a sufficient space for the carving of the boss, and then, by upwards and downwards projection, transferred to $k$ and $l$ to obtain their positions upon the upper surface.

The next operation is to obtain the section of the boss along the line me, and the same operation will give the vertical section of the lierne me. Draw mpen perpendicular to me. If the ridge rib de be level, ne will be equal to $D G$; if not, the given form of the ridge rib $D$ b will show the height of $E N$ as already explained.

Also MP is the axis of the boss, and will therefore be equal to $m k$, and for the same reason $\mathrm{p} p$ will be equal to $k \mathrm{k}$. The curvature of a lierne does not appear to have been in any case accurately determined. Sometimes they are made straight and sometimes a slight sweep is given to them as in the present figure, but which was evidently determined by eye merely.

For the joint $n q$, which connects the boss with the lierne, we must proceed by up and down projection as before, that is, place the point $c$ upon the plan so as to clear the ornamental boss, project this upwards to $n$ by a perpendicular upon me. Draw the joint $n q$, and project $q$ downwards upon the plan to $m$, which will be the seat of this joint upon the upper surface of the boss stone. The joint $n q$ is not necessarily directed to the centre of curvature of the arc NP in a lierne, as it would be in a rib. nqrs is a section of an actual lierne from Canterbury, in which the joints $n q r s$ at the two extremities are disposed at a small angle with each other, so as to prevent the stone from slipping down; but the joint $r s$ is nearly vertical, and $n q$ therefore very little inclined. In fact as the bosses at $P$ and $N$ are each firmly held up by their own ribs or arches AD AE, and the lierne acts merely as a strut or stay between them, there is no necessity to place the joints of the latter square to the curve, which in this and similar cases would either weaken the boss stone by cutting it back at $q$, or else make a larger stone necessary for it by carrying the point $n$ nearer to N . The acute angle in the lierne at $n$ renders the mouldings at that point liable to splinter at the joint, an effect that in truth may often be perceived in the practice of the age in question. I have already given a similar instance in fig. 5 (p.12). However, the joints of the existing lierne vaults will show that they were placed in the manner here exhibited.

For the section m $d$ of the boss, and for the lierne mp, proceed in precisely the same manner to describe the figure $Q \mathrm{R}$, in which $\mathrm{Q} v$, equal to $k K$, is the axis of the boss, and $w x$ the joint.

Thus we obtain the complete plan or mould $\operatorname{lm} k t$ of the horizontal upper surface of the boss stone, and may now proceed to cut it out of a given block as follows, having in the first place levelled its upper surface and transferred to it the plan lmkt.

The boss has four joints or beds, namely, at $l, k, n$, and $t$; and the vertical angles which these make with the upper surface are shown by the three sections of the boss which have been drawn, namely, those at $l$ and $k$ are $f h g$ and $h g e$, that at $m$ is $p q n$, and that at $t$ is $v w x$. By the use of a bevel, therefore, these beds can all be worked, and upon each of them must then be traced by a templet the outline of the mouldings of the rib. The distance of its lower edge is shown for each bed by the same sections which have given the angle; thus eg,fh,nq, and $x w$, are in each case the distance of the lower edge of the templet from the upper surface of the stone.

This done, the form of the stone can be completed without difficulty.
The key to this method is merely the employment of the horizontal upper surface, which being parallel to the plan of the vault, the respective ribs make with each other upon it the same mutual angles which they do upon the plan itself; and the vertical angles of the beds are also, as we have shown, immediately referred to the same surface. This surface I shall venture to call the surface of operation, because its sole use is to simplify the operation of shaping the stone, in opposition to the beds or joints which are surfaces of construction, or to the lower face, which is a surface of decoration.

That this method is in principle the one which was really employed by our workmen can scarcely be doubted after the examination of existing examples. The boss stone which has resulted from the diagram fig. 13 coincides in all its angles with the form of one which is (or lately was) lying amongst many others in the nave of Canterbury Cathedral, the ruins of a capital vault which was taken down together with Lanfranc's Tower, and which has already fur-
 bished us with another example in fig. $7^{18}$. Fig. 14 represents the plan and elevation of this boss stone, in which is

[^13]shown the horizontal surface of operation, still retaining the lines drawn from the axis of the boss in the direction of the respective ribs. Indeed, upon the upper surface of every vault which I have had the opportunity of examining when the boss stones are not hidden by rubble, I have always found that they had each the upper horizontal surface of operation, of the universal employment of which I have no doubt; for example, this is distinctly seen in the perspective view of the upper surface of the vault of St. George's Chapel at Windsor, fig. 32 (Plate IV.)

In explaining the above method, I have put it into a more compact and complete form than that under which it was probably employed. That the order of proceeding in the practical reasoning by which the form of the boss stone was elicited from the pattern of the vault, was virtually the same as that which I have given, can scarcely be doubted from the evidences that are still preserved to us by the horizontal surfaces of operation and the lines which they retain in the majority of existing examples.

In the following process I have retained the same general method, but have endeavoured to employ the least possible amount of descriptive geometry that is warranted by the remaining evidences. The actual process was perhaps intermediate between the two, or else the ruder method was used by one set of workmen and the more complete one by another set, which is the more likely to have been the case from the great difference of workmanship in the various examples that remain.

The simplest course of proceeding, then, that I can devise is this-draw the full sized plan of the ribs, liernes, and boss-circles, as in fig. 13; draw also the elevation of each rib, as нG, but not the elevation of the liernes, To work out any given boss stone, as m, mark upon the plan the place of the joints on the lower surface, at $a$ and $b$, and project these upwards to $e$ and $f$. Draw $e g, f h$, and the horizontal line $g h$. Level the block of stone to obtain the surface of operation, draw upon it the line which corresponds to the rib ad, and, having chosen a point upon this line for the axis of the boss $m$, draw lines from this respectively parallel to the liernes mf me. From the axis point set off the distances к $h \mathrm{\kappa} g$, obtained from the section of the boss eghf already drawn; and then, by means of the bevel, work the beds $h f g_{g} e$, the respective vertical angles of which to the surface of operation are given by the section.

Now with respect to the beds which receive the two liernes mf me, I have
relative position to all its neighbouring ribs and liernes in this plan as in the original, which is all that is requisite for the purpose of showing the identity of the methods which I propose with those from which the form of the boss was obtained.
already stated that these beds are not wrought in a direction tending to the axis of curvature of the lierne, as is necessarily the case with the beds hfge of a rib, which must be respectively directed to its axis L . The lierne beds, on the contrary, are wrought so as best to economize the boss stone. For these, then, it appears probable that no elevation of the lierne such as $r \operatorname{siq}$ was made, although this would have been the most complete method. We may therefore proceed to work these beds at an angle which approaches, as near as the stone will allow, to the required direction, assuming the latter by guess, and only taking care that the lierne bed shall intersect the surface of operation in a line drawn perpendicular to the plan-line of the lierne already drawn upon that surface. When the whole of the boss stones as well as the plain voussoirs of the ribs are finished, the ribs may be erected, and each boss stone will be of course held up in its proper place by the rib to which it belongs, and of which it constitutes a voussoir.

It now remains to construct the liernes themselves, for which purpose a vertical or side mould may be easily made of thin wood, by scribing for each lierne between its respective pair of bosses, now fixed in their proper position in the vault. Lastly, the pannel stones must be shaped and fitted, and thus the vault completed.

The difference between this method and the previous one is, that it dispenses with the up and down projection and the resulting face mould of the surface of operation. Also the liernes are shaped by trial instead of their forms being obtained by descriptive geometry. But the appearance of these liernes in many existing vaults, the irregularity in the curvature, length, and angular position of corresponding pairs of these members, which ought to be exactly alike in these particulars, as well as the apparently arbitrary position of their joints, all seem to show that their forms were determined by a trial of this kind after the bosses were fixed.

The only geometrical preparation that is here assumed is the full sized plan of the vault and the elevations of the ribs, and without these it would have been impossible to shape the boss stone so that its upper surface of operation should be horizontal. In fact, the universal presence of this surface is a sufficient evidence of the employment of the elevations at least of the ribs, if not of the liernes.

The employment of a boss stone, that is, of a stone placed at the intersection of two groin arches, and provided with beds for the reception of the four ribs that thus meet upon it, is not coeval with the use of groin ribs, as might be at first imagined. In the early Norman examples, one pair of the diagonal
ribs was first erected as a complete arch, and the other pair of ribs set up separately, and made to abut against the first arch, meeting either a voussoir or a joint, as the case might be. A very curious instance of this kind may be seen at the east end of the Norman crypt at Gloucester. The compartments of the aisle which runs round the circular end are necessarily trapeziums in plan. The two diagonal groin arches are each a semicircle in a vertical plane, and they consequently intersect, not at their crowns, but at a point considerably lower down. That no geometrical method had then been invented to obtain the form of the boss stone at the intersection is evident, for one diagonal arch is complete in itself, and the other, as above, being built in two separate halves, abuts awkwardly and obliquely against the first, having been evidently left to shift for itself, just as the vaulting surfaces were in the same early period, as I have shown in the beginning of this paper. In the round church of St. Sepulchre, at Cambridge, the compartments of the aisles are also trapeziums upon the plan, but the awkwardness of making the semicircular groin ribs intersect below their crowns, is very ingeniously got rid of. Those points of the outer wall from whence every pair of neighbouring ribs spring, are placed at such a distance from each other that the four feet of the diagonal ribs in each compartment really stand upon a rectangle, and the ribs therefore intersect at the crowns in the usual manner.

In the eastern transepts of Canterbury, there are boss stones with branches to receive the four diagonal ribs; but instead of attempting to work these branches to the same vertical curvature as the ribs they respectively receive, the branches are made straight and horizontal, and the ribs abut against them, forming a broken or discontiruous curve. Similar botchings may be observed in many early examples, and they are very curious as showing the necessity. for the geometrical system that afterwards arose.

In the boss stone $h f g e, f g .13$, the acute angle at $h$ together with the horizontat upper surface occasion a considerable waste of material, and make it necessary to employ a much larger block of stone for the boss than is required by its mere mechanical function. These acute angles are, however, always to be found in the early examples of complex lierne vaults, as in the Canterbury boss, fig. 14. Afterwards an improvement was introduced into the art of stone-cutting, by which part of this useless lump was got rid of, and the back of the stone worked vertically, making its section in the form egzyf. I shall reserve the explanation of this for the section on fan vaulting, to which it more properly belongs. The boss stones of the lierne vault of St. George's Chapel, fig. 32, are thus wrought.

Of the disposition of the liernes for the decoration of the vaults I shall say but little in this place, for the subject is sufficiently copious to form in itself a separate communication. I may, however, observe, that they are commonly so disposed as to make out a kind of star-shaped figure round each impost in combination with the ribs.

This in its simplest form may be seen in $f_{i g} .15$, where $\mathrm{A} d b e$ is one of the rays of the star. In this example, however, the star is double, another set of longer rays, as a $b c f$, being added. Also, in fig. 16 (p. 27), А в с Dem is the outline of a similar star, of which $G$ is the centre. I have elsewhere ventured to term these vaults stellar vaults, and have quoted continental examples of them, to which I beg to refer ${ }^{19}$.

Besides and in addition to these stellar forms round the springing of the vault, the liernes are so placed as to produce other ornamental figures about the centre of the vault. Thus, in fig. 16, the same series a bcde which forms the star round $G$, also represents the quarter of a cross with pointed ends, which has K for its centre ; and in fig. 19, a lozenge occupies the centre of each compartment; while in the vaults of the north-eastern chapel at King's College already described, a symmetrical six-rayed star is formed upon the crown of the vault by the liernes.

But my reason for at present alluding to this part of the subject is to point out two sets of examples, which in opposite ways serve to show the dependence of these designs upon geometrical constructions. Fig. 19 is the plan of a vault (from the monument of Archbishop Stratford in Canterbury Cathedral). In this plan the usual stellar form, radiating from H , appears to be abandoned, for the liennes gfedcba, which ought to constitute this figure in combination with the ribs, are so


FIG.19. placed that the regular pointed rays are lost by Fed and bсd lying each in one straight line. But when the vault is viewed in perspective (fig. 18), the stellar form appears, for $d e f$, which on the plan at defis in one straight line, is not so in the elevation, the point e being lower than F and d . The consequent depression of $e$ and of the similar point $c$, restores the figure of the star when the vault is viewed


FIG. 18. obliquely, but a person standing nearly under the middle of it, and looking upwards, would then perceive the regular crossed lozenge in the centre.

[^14]Thus the plan is ingeniously combined with the perspective effect to exhibit the two figures. In a great number of vaults of this class apparently symmetrical shapes are shown which entirely disappear or become distorted on the plan. Thus, in Canterbury nave, the vault in perspective appears to have a regular double star on each angle, like that of the cloister vault ( $f g .15$ ) of the same cathedral. But upon the plan this figure is so distorted that its existence would not be suspected. This argues a great power of foreseeing the solid effect of a design, and also a practical knowledge of the liberties that may be taken by throwing the plans out of symmetry without producing an appreciable distortion in execution.

On the other hand, in a large proportion of vaults, symmetrical lines and figures are introduced uselessly into the plan, for they disappear or become disagreeably distorted in the solid effect. The choir vault of Wells Cathedral is a very striking example of this, for its plan is a regular pattern formed of diagonal lines, with four rows of square compartments at their intersections, such as would form an excellent pattern for a flat ceiling, but being projected upon the surface of a groined vault, the symmetry of the figure, upon which from its nature its beauty alone depends, is entirely lost, the squares are distorted into irregular trapeziums, some of them bending round the groins, and the meaning of the whole destroyed. But this pattern must have been projected from the plan by a geometrical system, for the frame of the vault is very carefully constructed of ribs and liernes, and, as far as the mechanical construction goes, is a very admirable and intricate piece of work ${ }^{20}$.

As another and more curious kind of example, let us take the vault of Gloucester choir (fig. 17), of which two compartments are shown in outline. This vault has two sets of ribs, namely, principal


FIG. 17. and secondary, of which the first have more mouldings than the second. I have shown the principal ribs only in the compartmentebgm, and the secondary are added in aefg. The general form of this vault is a pointed waggon, of which нм is the ridge, and the transverse vaults of the clerestory windows enter this at a level considerably below that of the ridge.

These transverse vaults are shown upon the plan by shading. The curvatures of the ribs and liernes appear to be so carefully adjusted as to agree very

[^15]nearly with the surface of the great waggon vault at least. Now the plan of the primary ribs is evidently produced by drawing lines from each impost as at A to meet the centres of the ribs Ha and eg, and also the point G . But the rib A $n n p$ straight upon the plan, and, drawn in conformity with this system, completely loses its meaning in the solid effect; for it will be seen that it runs from a to the apex of the clerestory vault at $m$, then down again to the groin at $n$, and then up from $n$ to $p$ upon the surface of the great waggon vault. In execution this is an unmeaning line, and it is only with some trouble that its position upon the plan can be detected. In fact, the entire design exhibits in execution as little system as a spider's web, and yet, being erected of ribs and liernes with bosses at their intersections, must have been derived by careful projection from the plan as here shown, and the effect is after all rich and peculiar.

The vaults of the transepts of Gloucester, that of the north transept of Bristol Cathedral, and of St. Michael's Chapel at Canterbury, may be also cited as belonging to this class, as well as the now destroyed vault of the choir of York.

The mechanical construction of these lierne vaults is various. Most of them, at least the larger and simpler examples, are built exactly in the same manner as the rib vaults already described, that is, a stone framework is constructed of ribs, liernes, and boss stones, all of which are rebated for the reception of stone pannels or vaulting surfaces, which are dropped in from above. When the vaults are large and the ribs and liernes few, these intermediate surfaces are built of light rough masonry as before; but when the intervals are made smaller by the increased number of the reticulating ribs and liernes, the connecting surface becomes a mere pannel of one or two stones, each of which was, as I suppose, scribed, as a carpenter would call it, to fit the frame.

Fig. 15 (Plate 1.) is an example of this method, from the cloister of Canterbury Cathedral.

The span of this vault is thirteen feet, the ridge ribs are level, and it approaches in effect very near to a fan vault, although it is not one, as I shall show when I come to the explanation of that class of vaults. In fact the middle plans of the spandrel, instead of being circles round the impost a as in fan vaults, are of the figure bcd. The tas de charge, or portion of masonry above the impost, which contains the nascent ribs in their state of entanglement, is laid in three courses, of which the lower is two feet six inches in thickness, and the others one foct three inches, and nine inches respectively. The
upper stone has inclined beds formed for each rib to spring from, exactly as I have already described the earlier vaults (page 7), and the plain portions of rib which connect this stone with the boss stones in the crown of the vault are built of voussoirs about eighteen inches more or less in length.

The bosses are so numerous, and lie so close together, that the stumps of ribs that are in one piece with adjoining ones touch for the most part, as shown in the drawing, with one or two exceptions, where small thin slices of rib are interposed; these are marked $m$ in the figure. Some of the compartments are foliated, but the cusps are attached to and worked out of the boss stones. The pannels are laid in in one or two pieces, as the case may be. Their joints, which occur irregularly, are indicated by dotted lines.

In this and similar examples the stumps of ribs which spring from each boss stone are worked separate as so many branches. This is shown at the margin of the figure at P and Q . In other vaults of this class, however, the boss stones are so worked that they each contain a portion of the pannel surface between the short ribs. This is the construction of the vault fig. 16 (p.27), from the gateway of Queen's College, Cambridge. Each boss includes between its branches as much pannel surface as is contained by a straight line drawn from one to the other; but for the rest the construction is the same as the above. Adjoining bosses either touch, as в and $c$, or are connected by short pieces, as $\mathrm{M}, \mathrm{N}$; the remaining portion of the intermediate surface, as $P, Q$, is a pannel dropped in from above. The lower portion of the ribs and their connecting surfaces are precisely constructed as in the previous examples.

In a third construction, however, the rib and pannel method is entirely abandoned, and the vault, the same in decorative appearance as the previous ones, is nevertheless built of solid jointed masonry, every rib stone carrying with it in one piece the requisite portion of the neighbouring intermediate surface. An excellent specimen of this mechanical construction is the vault of Heury the Fifth's tomb at Westminster; but as this method appears to have grown out of the fan vaulting, I shall proceed to that, and postpone the description of the vault in question for the present. There is also a good specimen of this construction in the umbrella like roof of a staircase which leads to the top of the small south-western tower of Peterborough Cathedral.

It thus appears that the rib and pannel construction was finally driven out and superseded by solid masonry, although to all appearance the vaults continued to be formed of ribs and pannels as usual. Very early in the history of vaulting, the lower extremities of the pannels were made solid, as I have shown in page 7. Next, it was found simpler and stronger to work the small.

portions of pannel surface between the branches of the boss stones out of the same block, than to cut them away and drop in the pannel afterwards. Then the bosses, from the increasing complication of the patterns, began to approach so close, that it was worth while to take the pains to make them meet, and thus the crowns and ridges of the vaults came also to be built solid. Lastly, the solid construction was extended to the entire vault, and so by gradual degrees the mechanical and decorative construction of the vault, which began by being identical, ended by becoming totally different.

Section IV.-On Fan Vaults.

In fan vaults, the difficulties of adjusting the curvatures of the ribs are at once disposed of by making them all of the same curvature. We may suppose that the increasing complication of the patterns in the vaults of the kind already described, and the great number of intermediate ribs and liernes that were thus introduced, may have led to the invention of this method, by which certainly the number of templets and the difficulties were greatly diminished. The construction of these fan vaults is in all examples so nearly the same, that they seem to have proceeded from the same workshop; and it is remarkable that, at least as far as I know, there are ao continental examples of them, whereas of ihe previous vaults there are quite as many on the continent as in England. In France, indeed, the lierne vaults are not very numerous, they are confined to small chapels, and their patterns are in general simple; but in Germany and in the Netherlands there is an abundance of them, distinguished certainly from ours by local peculiarities, but nevertheless of similar mechanical construction, and requiring the same geometrical methods.

The general construction of fan vaults may be understood from figs. 24, 25 (Plate I.), and 26 (Plate II.) These represent one of the most complete, which covers a kind of chapel at the east end of Peterborough Cathedral. The workmanship of this vault is the most perfect of any that I have examined. The span of King's College Chapel (forty-two feet) is much greater, but its workmanship is not nearly so good. The span of the Peterborough vault is twenty-six feet from wall to wall.

The pattern of these vaults is always a kind of tracery of the same class as the perpendicular tracery which at that period occupied the heads of windows or the surfaces of walls, the only difference being that the mullions, instead of
being parallel and vertical, are curved and radiate from an impost. Instead, therefore, of the horizontal transoms of windows, we have level circles intersecting the ribs at nearly equal distances, and intermediate rays or ribs are inserted between the principal ones. The pannel spaces into which the surface of the vault is thus divided are furnished with arch heads and foliation, exactly as the similarly produced pannels are ornamented in the tracery of windows.

I have already stated that in the lierne vaults a star pattern is commonly found, which radiates from each impost in the same way as the fans of the present class of vaults. The difference between the star and the fan is, that the star is formed of ribs, which may be and often are of different curvature, and the rays of the star of different length, but so skilfully arranged with regard to their solid effect, that although irregular upon the plan, this irregularity is not offensively perceived in the complete work.

The fan is, on the other hand, formed of ribs strictly of the same curvature and elevation, and its outline is bounded by a horizontal circular rib instead of the zig-zag line of liernes in the former vaults by which the rays of the star are pointed. The effect of the fan is that of a solid of revolution upon whose surface pannels are sunk; the effect of the star is that of a group of branching ribs. This difference of character is assisted by the mode in which the mouldings are disposed in the two cases. In the older vaults a vertical plane upon the plan line of every lierne or rib divides it into two symmetrical halves; but in the fan vaults the moulded rings and arch heads are all perpendicular to the surface of the vault. This will be more fully explained presently.

In fan vaults the quantity of decoration is so much increased, and its parts become so small, that it is no longer practicable to frame the tracery of these vaults on the rib and pannel system with liernes and boss stones, and consequently the portions near the crown of the vault, where the decoration is compressed and crowded, are always constructed of jointed masonry. But the branching ribs below the first series of arch heads are still built of long stones filled in with pannels, and the tas de charge from whence the ribs spring is also laid in level courses as before. There are a few exceptions to this general description ; for example, the vault of Henry the Seventh's Chapel, which, however, differs in many other particulars from these fan vaults, and will be separately described. It is entirely constructed of jointed masonry without pannels. Also the vault of Islip's Chapel at Westminster is a real fan vault, but without any mixture of rib and pannel work.

In fig. 24 the upper half of the plan represents the lower surface of the
vault with the joints of the masonry and pattern of the tracery, and the lower half shows the upper surface of the vault with its joints. Fig. 25 is a vertical section through a c, and fig. 26 an isometrical projection of the upper surface. The portion of vault included within the spandrel ABC, consisting of plain ribs only, is constructed of rib and pannel work exactly like the older vaults; but all the remainder of jointed stonework, as shown in the lower plan. In the King's College vault, and in many others, the short straight portions of rib that lie between k and L are also constructed in separate stones with pannels dropped between ${ }^{21}$.

The section shows that the spandrels of this vault are filled up to the level of E with rubble work, as are also the spandrels of King's College vault. In addition to this, a wall $x y$ is at Peterborough added to stiffen the spandrel. This I have not seen elsewhere. Every one of the stones of the jointed masonry retains a portion of the horizontal surface of operation, which in this construction is evidently employed for the same purpose as in the boss stones of the lierne vaults. These horizontal surfaces are indicated upon the plan by the light tint, and are also shown in the section at $g, h$, and $k$, and more clearly in the perspective drawing, fig. 26.

The joints of the masonry are evidently arranged with reference to the pattern of the vault. Thus a ring of stones round a beginning at в к is made to contain all the first series of pannel heads, with the circular rib and crown of Tudor flowers. The succeeding ring from k m contains the next series of pannels, with their arch heads and the Tudor flowers above therr. As to the next ring, it begins at $\mathrm{m} N$, by extending across the crown of the arch; and when the stones become too large, they are divided, as at $m n, p q$, and $r s$, in such a manner as to interfere as little as possible with the lines of the pattern. It is remarkable that the masons of this period often place a joint in the very midd̃le of a projecting rib, as at $p q$. As to the radiating joints, they lie in this example in the middle of the pannels, but in others not universally so. Since the disposition of the joints is thus governed entirely by the pattern of the tracery, it is clear that in setting out the moulds of the stones the lower joints must first have been drawn upon the plan, and then the moulds of the upper surfaces obtained from them by a system of projection similar to that which I have already applied to the lierne vaults and bosses.

It must be supposed that the lower surface of the stones was worked smooth, and the lesser ornamental details drawn upon this surface, so that,

[^16]for the purpose of settling the places of the joints, only the ribs and circular bands were drawn on the plan; for if the entire pattern were drawn on the plan and transferred to the surface of the vault by projection, as in the case of the complex lierne vaults, the arch heads, circular pannels, and other figures which decorate the spaces between the principal lines would be distorted after the manner of the squares in the choir vault at Wells, already mentioned. But such distortions never appear in these figures in fan vaulting, so that we may assume that these details were traced upon the lower surface of the vault stones, but that all the principal lines necessary for obtaining the form of the stones and distribution of the joints were projected from the plan. Indeed the new manner in which the mouldings are worked, namely, perpendicular to the surface of the vault, as already mentioned, shows that the patterns were now drawn upon and sunk into that surface, and not derived from the plan.

I will now explain the steps by which I suppose the shapes and moulds of the stones to have been obtained, and which in principle is the same as the method by which I have obtained the boss stones and liernes, namely, by means of the horizontal upper surface of operation, of which there is abundant evidence to prove the employment in all the existing specimens of fan vaulting of which the upper surface is accessible.

Fig. 21 explains this method, and corresponds in the plan of the vault to fig. 24 (Plate I.) ; the examples chosen being the stone marked P and those which correspond to Q and R on the opposite side of the ridge rib.

On the plan fig. 21, the ribs and circular bands being inserted, as shown by the double lines, draw a $B$ the central line of the rib upon which the stones in question lie, and upon it draw in elevation c D , the curve of the rib, which, from the nature of fan vaulting, is the same for every rib.

The joints at the lower surface must next be drawn upon the plan, as shown by the dotted lines, due regard being had to the ornamental details which are to be subsequently added. These joints are either radial or circular, from the nature of fan vaulting. The radial joints which converge to the centre a are all in vertical planes, and therefore their places upon the upper plan or surface of operation will be vertically above and coincident with their places upon the lower surface, as at $a b$ and $c d$. But the circular joints are arch beds ${ }^{22}$, and their places upon the upper surface may be obtained from those of the lower surface by upwards and downwards projection as follows.

[^17]The circular joints intersect the line $A B$ at the points $n, q, s$. Project these upwards to meet the curve cd at $N$, $Q$, $s$. Draw fmx concentric to the curve, and at a distance from it equal to the necessary thickness of the rib co.


FIG. 21.
From the centre of curvature of c d draw the arch joints n m, Q P, SR, intersecting their outer curve at $\mathrm{M}, \mathrm{P}$, and R ; and through these points draw vertical lines mт, $\mathrm{P} V, \mathrm{RW}$, also projecting the same points downwards to the plan at $m, p$, and $r$. With centre a draw circular arcs $a m, c p, g r$. Thus will be obtained the plans or moulds of the upper level surface of operation of the stones $\mathrm{NP}, \mathrm{QR}$, and the horizontal lines TP VR will be the sections of these apper surfaces.

To complete the third stone sx , however, it is necessary to draw the curve of the ridge rib; for this stone, being placed at the meeting of two ribs ab fb, and of the ridge rib abh, resembles in this respect a boss. It is in fact upon the plan a six-sided figure, bounded by four arch joints, $g t, t w$,
curve CD; and MN being any given arch joint of this solid, produce it to meet the axis in $z$. The bed of which $M N$ is a section is a frustum of the cone, that would be generated by the revolution of $Z N$ round the axis A $z$.
$w x, d y$, and two radial joints $g d, y x^{23}$. The joint $w x$ being opposite and similar to $g t$, the same section wsxd will serve for the two joints, but the joints $t w, d y$, must be obtained from the elevation of the ridge rib, which may be drawn as follows.

Since the arch ribs of a fan vault are all of the same curvature, the ridge ribs can no longer be horizontal, and their form is in fact determined by the intersections of the arch ribs, so that when the dimensions of the parallelogram which is to be covered by each severy of the vault, and also the curve of the rib, are given, the form of the ridge rib may be found. For example, as follows:-Letabcd, fig. 20, be one quarter of the severy, adagafaeac the ribs branching from A , of which $a \mathrm{KNP}$ is the curve in elevation upon $A B$. $A C$ is made equal to the longest rib, namely, the diagonal rib, Ac; and the lengths of the other ribs are transferred by arcs of circles round the centre a to the points $d g f e$ respectively. Draw perpendiculars upon these points meeting the curve in klme and $P$, then will $G$ be the plan of the point at which the ribag intersects the ridge rib, and $\mathrm{A} g$ is equal to $\mathrm{A} G$, consequently $a_{\mathrm{L}}$ is


FIG. 20 . the portion of arch curve occupied by the rib $A G$, whence $g_{L}$ is the height of the ridge rib at $G$ above the impost plane. In like manner $\mathrm{A} e$ (equal to AE ) shows $a$ N to be the quantity of arch curve occupied by the rib AE, and thence $e \mathrm{~N}$ is the height of the ridge rib at e. Similarly it can be shown that $d \mathrm{~K}, g \mathrm{~L}, f \mathrm{~m}, e \mathrm{~N}, c \mathrm{P}$, are the respective heigbts of the points $\mathrm{D}, \mathrm{G}, \mathrm{F}, \mathrm{E}, \mathrm{C}$, of the ridge rib. Draw, therefore, perpendiculars $\mathrm{D} k, \mathrm{G} l$, $\mathrm{E} m, \mathrm{e} n, \mathrm{c} p$, through these latter points, producing them to A B , and also parallel lines $\mathrm{k} k, \mathrm{~L} l, \mathrm{~m} m, \mathrm{~N} n, \mathrm{p} p$, intersecting them in $k, l, m, n$, and $p$, then will the curve drawn through these points to the elevation of the ridge zib referred to $A$ b.

The curve so obtained being transferred to the plan fig. 21 at $g h$, the section of the stone in the middle plane of the ridge rib may be drawn as

[^18]at $\kappa$, by upwards and downwards projection, in the same manner as the other sections have been obtained.

In this stone the perpendicular bdxupon the intersection of the ribs corresponds to the axis of the boss in fig. 13 (p. 32).

The stone NP may now be worked as follows. Having levelled the upper surface of operation, trace upon it the upper mould which has been described upon the plan at $m a c p$, and also the arc $b q$. Work the radial joints perpendicular to this surface, and trace upon them the side mould nмтpq. Work a concave surface also perpendicular to the surface of operation, and meeting it in the line $a m$, and a convex surface meeting it in the line $b q$. Fig. $21 a$ represents the stone in the state to which it has now been brought. It is viewed angularly; $t_{\mathrm{TP}}$ is the edge of the surface of operation, NMTPQ one of the radial sides plane and perpendicular to the latter surface, and $\mathrm{T} t \mathrm{~m} m$ is the concave surface, also perpendicular to the upper surface. The outline of the side mould is shown traced upon the radial surface. To complete the stone, it remains to form the two conical arch beds $\mathrm{m} m_{\mathrm{N}} n$ and PQ . The latter is easily wrought, by tracing a horizontal line on the convex surface, beginning at Q , and then working away the stone in straight lines from this line to the arc already traced on the upper bed from $P$; or this bed may be worked by means of a bevel set to the angle TPQ, and thus the necessity for working the convex surface be dispensed with.

In like manner the lower arch bed m in $\mathrm{N} n$ may be wrought in two ways: first, by tracing from $m$ the horizontal line $m m$ on the concave surface, and working the bed by a bevel set to the angle тмn; secondly, by working the lower surface of the stone into a horizontal plane passing through $N$ and parallel to the upper surface, tracing the arc $\mathrm{N} n$ upon the lower plane, and then working away the stone in straight lines from $\mathrm{m} m$ to $\mathrm{N} n$. The bevel method, being the simplest, is most probably the one which was employed by the Middle Age workmen. The other method is more analogous to modern practice, and is susceptible of greater precision. The two arch beds being completed, the lower surface $n \mathrm{NQ}$ is easily wrought from the lower boundary $\mathrm{N} n$ to the upper boundary which passes through Q , by means of a sweep or templet whose edge is cut to the curve NQ .

A similar process will give the form of the more complex stone at the intersection $\boldsymbol{в}$ of the ribs.

In forming the boss fig. 13 (p. 32), the stones were worked to an acute angle as at $g h f$, by which means a superfluous angular lump of stone is left at $h$. If the upper plan of the stone had been projected as in the above method
from the point $y$, where the arch joint $f h$ meets the outer curve of the rib, the useless portion $y h \mathrm{z}$ would have been saved, and a smaller stone have served the purpose. This is a more complete and scientific process; and while the angle $g h y$ remaining in the Canterbury bosses and others is evidence of the ruder process, the vertical surfaces тмmt ( $f \mathrm{fg} .21 \mathrm{a}$.), which may still be traced in the fan vaults, prove also the use of the improved method which I have just explained. In the later lierne vaults, however, this latter method is also applied to the bosses. In fact the fan vaulting was invented long after the lierne vaulting; but they continued to be practised together up to the latest period; and the improved system of stone cutting introduced by the increased complexity of the fan vaults would naturally be applied to the lierne vaults. Thus it happens that many of the later specimens of the latter are constructed with solid masonry instead of the original rib and pannel method. In the view of the roof of St. George's Chapel (Plate IV.), it will be seen that the bosses were worked vertically downwards from the plan in the same manner as the stones in $\operatorname{jg} .21$.

The arch joints or beds of bosses, however, are planes, but those of fan vaults ought to be conical surfaces. This is not universally the case in practice. The joints of the vault of Henry the Seventh's Chapel appear to be in many parts of the severy worked in planes. Those of Islip's Chapel are also worked in planes, as shown in $f i g .22$. This is a mere diagram of a portion of the fan, and therefore the joints are not projected over the mouldiags.

The fans are in this example constructed entirely of solid masonry, and not of rib and pannel work. $A B$ is a wall rib, $A D$ a diagonal rib, $A C$ an intermediate rib, and the arch joints are planes, arranged with respect to the ribs $A B A \subset A D$ so as to pass respectively through the axis of curvature of each of these ribs respectively. In other words, if the vault be divided by lines a $b$ a drawn through the middle of each compartment, every portion which is contained between two of these adjoining lines $b_{\text {a }} c$ a forms an ordinary arch with plane joints. But in the lower stone or tas de charge the joint line runs obliquely across the tierceron, and beds are only provided for the wall and diagonal arches. Similar plane arch joints may be found in some parts of the smaller vaults at Peter-
 borough ; but the curvature of joints of this kind differs so little from a straight line, that it is not always easy to satisfy oneself which of the two is the case, unless the joints are very accessible and decidedly marked, as they happen

to be in the Islip Chapel. In the King's College vault the joints are circular, and the arch beds consequently conical.

In a similar manner the pannel surfaces of fan vaults which ought to be convex, and are so in many specimens, are wrought in planes in others, although the horizontal ribs are made circular. But in the fan vault of Wells tower and Islip's Chapel the horizontal rib runs in straight lines from one arch rib to the other, thus giving polygonal faces to the fans, and in these the pannel surfaces are planes of course.

Fig. 23 shows the arrangements of mouldings which have been already alluded to as prominent characteristics of the two kinds of vaulting. In a в a rib is shown, which is intersected in its course by two liernes at $a b c d$. The middle plane of these liernes being as usual vertical, it is plain that their mouldings cannot mitre (as it is termed) with those of the rib, because they are placed symmetrically to their own axis, while the mouldings of the rib cross at an angle which throws them on one
 side below, on the other above, the corresponding ones in the lierne. This is shown more distinctly in fig. 14 (p. 35), and leads to great difficulties in the management of the intersections of these mouldings, which are in fact generally covered by bosses that prevent their awkwardness from being discovered; and we might fancy that it was for this purpose the bosses were invented, were it not that they are employed for the irtersection of the crowns of arch ribs, in which no such difficulties occur, long before the liernes are introduced. The horizontal ridge ribs, however, which are much older than the liernes, present the same kind of difficulty in their intersection with the crowns of pointed arches. To overcome these difficulties, we sometimes find, when bosses are not employed, that the mouldings of the rib are awkwardly warped and distorted at their junctions, as from $d$ to $b$; also that the plain side of the lierne is several inches higher on one side than on the other, as at $a$, or else that the surface of the vault is dropped on one side to accommodate the lierne, but which introduces a similar difficulty with the rib, as at $c .{ }^{24}$ Nevertheless the branching and independent character of the ribs and liernes is so intimately bound up with this verticality of their middle planes, that I cannot help ascribing one of the first departures

[^19]from the true Gothic character to the change in the method of intersecting the mouldings, which is the leading feature of fan groining.

This is shown at $\mathbf{d e}$, where a rib of the same profile as a в is intersected by two others, $f g h k$, and by placing their axes of symmetry or middle sections perpendicular to the surface of the vault, a perfect mitring of the mouldings is obtained, and the workmanship of these mouldings is at once greatly simplified and reduced to the capacity of a much lower class of workmen. But thus the ribs and other moulded lines become at once visibly referred to the general surface of the vault and subordinate to it, decorating it nearly after the same fashion and principle as the pannels and concamerations of a Roman vault, which are in like manner sunk perpendicularly to the general surface; whereas, in a genuine Gothic vault, the real surface should be subordinate to the branching ribs, and rest upon them.

In some of the lierne vaults subsequent to the introduction of fan vaulting, the liernes are worked perpendicular to the surface of the vault, as for example in the porch of Bishop Booth, at Hereford (1516-1535).

The practice of thus working the mouldings and ribs square to the surface may possibly have originated with the makers of shrine work and small canopies, which are often ornamented with very elaborate models of vaults. These are beautifully ribbed into patterns, and as the entire vault is worked out of a single stone, the mouldings are naturally sunk with reference to its surface. In the larger canopies of monuments, tou, a similar method is often followed. Thus the monaments of Archbishops Stratford and Sudbary, in Canterbury Cathedral, have canopies vaulted with small lierne vaults very nearly alike, but in the former the mouldings are vertical, and in the latter square to the surface. Stratford died in 1348, and Sudbury in 1381.

In all the great fan vaults, the surface of operation is reduced by cutting away a considerable portion of the stone, so as to bring it to the form which is shown at $f g .21 b$, and also in the isometrical view $f g .26$. This is effected by removing the portions marked E in fig . 21 . The remaining part of the surface of operation retains lines drawn in the direction of the ribs, as shown in fig. $21 b$, but which are not inserted in the other drawings of the complete vault, to avoid confusion between them and the joint lines.

This portion of the stone appears to have been removed to lighten it, for from the rude and irregular manner in which it is cut away, and from the remains of the vertical surfaces тм $t m$ at the lower parts of the stones, it is clear that they were first worked completely into the form shown in fig. $21 a$.

Vault of Henry the Seventh's Chapel, Westminster.--In this example the surfaces of operation are entirely chipped away, and the upper surface of the pault reduced to parallelism with the lower; but in one or two places these surfaces remain, apparently forgotten, and traces of them are seen in many others, so that they certainly were employed there.

The mechanical structure of this vault is different from that of the other great fan vaults, for a great arch is thrown across between each severy, and each principal fan springs from one of the voussoirs of this arch at a considerable distance from the wall. Thus each fan, instead of including, as in the usual method, half the solid of revolution, contains the entire solid, at least at the lower part. These fans are, however, met by others, which spring from the wall in the usual manner. The same principle of construction had been adopted in the vault of the Divinity School at Oxford, said to have been finished about $1480^{25}$. This is the earliest instance which remains at present, for Henry the Seventh's Chapel was founded in 1502. The vault of the choir of Christ's Church, Oxford, by Wolsey, about 1528 , is also upon a similar principle.

Fig. 27 (Plate I.), A BCD is a plan of half a severy of the vault. The upper half of the plan shows its lower surface with the ribs and pannels, and the lower half of the plan shows the upper surface of the vault and the joints. Fig. 28 is a vertical section through a line $c d$ close to the face of the great arch.

The great arch feg springs from the walls at F , the joints below f being horizontal in the usual manner, and the portion Fg lies below the vault and is visible from below. A branch arch at $н$ again connects it with the walls, and the space between the latter and these arches is also filied up with tracery, all which serves to stiffen the free portion of the arch, and prevent it from giving way by curving inwards between F and $g$. At $g$ the arch pierces the surface of the vault, and the upper portion of it EG lies above that surface, its position only being marked on the lower surface by a continuation of the hanging tracery or flowers which decorate the portion Fg .

The voussoir e, which also carries the pendant $k$, is a very large stone, connected by the joints $e f$ and $g$ with the rest of the structure. $f$ and $g$ are the arch joints, by which it is united to the great arch; but there is also a conical surface or bed extending from $e$ to $f$, and shown on the plan at $m n p$, from which the masonry of the principal fan or conoidal vault radiates upwards and outwards in all directions, until it meets the neighbouring fans along the lines DMMN and NP. A fan vault of the same section rises from a conical

[^20]bed $r s t$, formed upon the surface of a stone $c$, which projects from the wall. This vault meets the neighbouring ones along the lines Pn and nr.

The rib and pannel method is wholly abandoned in this vault, which is entirely constructed of jointed masonry, as shown in the plan. This is disposed in concentric rings round the centre of each fan, and the radiating joints are placed so as to break joint with each other, that is, so that the joints of one course fall opposite to the solids of the next course throughout. All the joints are arranged in relation to the pattern, sometimes running along the middle of the ribs so as to split them. At $v w$ and $x$, the voussoirs of the great arch are made to carry the neighbouring portion of the pannels. As before mentioned, the upper surface of this vault is worked into parallelism with the lower. This gives to the upper surface a very striking and bold effect, by displaying most plainly the entire mechanical construction. Its appearance is shown in fig. 29 (Plate III.), which is an isometrical drawing of the vault, and which also represents it as having some portions dissected away, to show better its relation to the great arch below. The vault is very thin. The pannels vary in thickness from three to four inches, and the principal ribs project eight inches from the pannels. The great arch rises seven inches above the upper surface of the vault, and is twelve inches wide.

The fans of this vault, in fact, as far as the stone cutting and the disposition of their joints are concerned, differ in no respect from a dome of masonry, the rib and pannel method having entirely disappeared; for a dome like one of these conoidal vaults is a solid of revolation, and like it would be built of concentric courses with breaking vertical joints. The forms of the stones, however, are easily obtained by the use of the surface of operation, as already explained in the case of the common fan vaults.

I have said nothing respecting the chronology of the vaults which are the subject of this paper, wishing to confine myself entirely to the question of form and construction. The history would have led me into discussions beyond the limit of this paper, and would interfere with its purpose. However, the fan vaults form so definite a group that they present much less difficulty in this respect than the others, and I have therefore thought it better to add a few remarks upon this subject.

The subjoined list contains all the principal examples of which I am aware, and most of the smaller ones. They are arranged as nearly as possible in the order of time. The column headed Span, contains the longest side of each severy in round numbers to the nearest foot, for the purpose of showing the comparative magnitude; but as the dimensions are mostly taken from engraved
os. Kmg : I I

plans, I have not specified those dimensions in inches. It must be also understood that the list contains only genuine fan vaults according to the definition given above, namely, that the fan shall be bounded by a horizontal rib. This excludes some vaults that approach very near to fan vaults, but are in reality stellar vaults, such as the Divinity School at Oxford, and West's Chapel at Ely. In two of the examples, however, the central tower at Wells, and Islip's Chapel, the fans are bounded by horizontal polygons instead of circles. The dates in the table are those usually assigned.

| $\underset{\text { Span }}{\text { Sin feet. }}$ | Example. | Date. | Reference to Engravings. |
| :---: | :---: | :---: | :---: |
| 12 | Cloisters, Gloucester..........................(s.) | 1381-1412... | Britton's Cathedrals. |
| 8 | Inner Porches, Gloucester | 1420-1437 |  |
| 7 | Vault in All Souls, Oxford ..................(s.) | 1437-1444 ... | Pugin's Examples. |
| 8 | Vault in St. John's, Oxford .................. (p.) | 1437-1444 ... |  |
| 10 | Beaufort's Chantry, Winchester ............(s.) | about 1445 .... | Britton's Cathedrals. |
| 18 | Dean's Chapel, Canterbury ..................(s.) | 1449-1468 ... |  |
| 8 | Chapels of Lady Chapel, Gloucester ............ | 1457-1472 |  |
| 8 | Stanbury Chapel, Hereford ..................(s.) | 1453-1474 ... |  |
| 12 | St. George's Chapel, Windsor, side aisles ...(s.) | after 1475 | Britton's Antiq. |
| 37 | Central tower ...........(p.) |  |  |
| 9 | Waynflete's Chantry, Winchester ............ (p.) | 1486 | Britton's Cathedrals. |
| 15 | Alcock's Chapel, Ely..........................(s.) | 1488 | Bentham or Miller's Ely. |
| 11 | Islip Chapel, Westminster | about 1500. | Neaie's Westminster. |
| 31 | Bath Abbey Church ..........................(p.) | founded 1500... | Britton. |
| 34 | Henry VII. Chapel, Westminster............(p.) | founded 1502... | Coitingham or Neale. |
|  | Chapel in Cirencester Church. | 1508 (upon the vault)........ |  |
| 25 | Central Tower, Canterbury....................... | 1495-1517 ... |  |
| 26 | East end of Peterborough Cathedral........(p.) | 1440-1541 | Briton's Cathedrals. |
| 14 | Audley Chapel, Hereford.......................... | 1492-1525 |  |
| 44 | King's College Chapel, Cambridge...........(p.) | 1513. | Britton's Antiq. |
| 7 | Redmount Chapel, Lynn......................(s.) |  |  |
| 18 | Hampton Court Gateway .....................(s.) | after 1520. | Pugin's Specimens. |
| 18 | St. Laurence Chapel, Evesham ...............(s.) | 1513-1539 | Rickman's Gothic Arch. |
| 8 | Salisbury Chapel, Christchurch, Hampshire (p.) | 1540 | Ferrey's Christchurch. |
| 10 | Cloister, St. Stephen's, Westminster.........(s.) | 1526-1547 ... | Britton's Houses of Parl. |
| 30 | Central Tower, Wells. <br> Staircase, Christchurch, O | 1640 |  |

The cloister of Gloucester is the earliest example, but the date of this is rather uncertain. The work is usually attributed to Abbot Froucester (1381 -1412), but apparently upon no better foundation than tradition. Leland says, "These notable things following I learned of an ould Man made lately a Monke of Gloucester. . . . Abbot Froucester made the Cloyster, a right goodly and
sumptuous peice of Worke" . . (Itinerary, vol. iv. fol. $172 a$ ). In the account of this cathedral in the Addenda to the Monasticon (vol. i. p. 995), the cloister is mentioned, but not assigned to any founder. It was probably carried on gradually through a long period, as it exhibits changes of style. There are also at Gloucester two examples of designs for fan vaults of remarkable construction. First, the vaults of the north and south chapels, which are annexed to the Lady Chapel, founded by Abbot Hanley (1457-1472). These are alike, of small span (eight feet), of excellent workmanship, and are genuine fan vaults, but the fans abut against a flat ceiling, decorated with tracery, in a row of wheel-like compartments, which extend from one end to the other of the vault, and also between the fans themselves laterally. The tracery of the fans is also different from that of the other specimens of the list, being much richer and more elaborate. Secondly, the great western door and the north-eastern door of the cloisters in the north side aisle of the nave are furnished with internal porches, in which the principle of fan vaulting is applied with great ingenuity. The door is a folding door in two leaves, and its upper edge is cut to fit the head of the archway which receives it. A quarter-fan on each side constitutes the vault of the porch, the fans each springing from a single shaft. The hinges of the doors are let into these shafts in such a manner that the door when opened revolves round the shaft as an axis, and consequently the arch-shaped head of the door exactly circulates about the fan which it fits, and from every point of whose surface its edge is at a small equal distance in all positions. The sides of the porch are also exactly of the same shape as the leaves of the door, so that when the leaves are opened they may be laid flat against these sides, which they exactly fill. The consequence of the employment of quarterfans is plainly that the inner opening of the porch is square-headed, and this is filled up by a four-centred arch with open tracery in the spandrels. The entire composition is a most original, and, as I believe, unique application of this species of vault to the mechanical contrivance of a door; for whether the door be open or shut, it finds an exact recess to receive it, and in its motion about its hinges its upper edge exactly represents the geometrical generation of the solid of revolution which is employed for the vault above it ${ }^{26}$.

The description of a fan which I have given in page 44-that it resembles

[^21]a solid with sunk pannels rather than a ribbed vault-is more prominently the case in these Gloucester vaults, the earliest of the series, than in their successors; for the only rib mouldings that rise from the abacus are the wall ribs and the transverse rib, and the remainder spring from arches which branch from these in succession, forming pannels, and completely depriving the vault of its proper decorative construction. This management gives a much greater appearance of novelty and originality to these specimens compared with lierne vaults than even the later fan vaults possess, and makes it the more probable that they were the first fruits of the new contrivance.

But indeed, through all the perpendicular portions of Gloucester Cathedral, there runs a vein of invention that marks them as the work of a series of bold and original geniuses, any one of whom were capable of making so great a step in masonry as the invention of fan vaulting, the more especially as the elaborate designs of their lierne vaults, and the general management of the mouldings and tracery of this building, show the masons to have been perfect masters of the art of stone cutting, and to have sought out all occasions to display their power over it in quaint conceits and intricate interlacings and combinations of form. I know the danger of indulging in such speculations, but I think it is not too much to suppose that a peculiar school of masonry was established in connection with this building, and to this I should be well content to assign the invention of fan vaulting.

On the whole, we may fix the introduction of fan vallting in the first, quarter of the fifteenth century. The first work of considerable span is the Dean's Chapel at Canterbury annexed to the north-west transept; this is the known work of Prior Goldston the First, who was elected in 1449, and died in 1468. In the Obituary, the chapel is described as having a stone vault of elaborate construction: "Capellam cum testudine lapideâ valde artificiosâ." (Wharton's Anglia Sacra, vol. i. p. 145.) The work of St. Gecrge's Chapel at Windsor proceeded through a long series of years, and its principal vault is not a fan vault; but the side aisles have small fans, aind the central compartment or crossing also has a genuine fan vault of large size, erected in $1528^{04}$. The great works of known date, namely, the central tower of Canterbury, King's College Chapel, Henry the Seventh's Chapel at Westminster, and Bath Abbey Church, are all about 1500.

In many of these vaults the plan of each severy is an exact


FIG.20.b. square. In this case the fan is a perfect quadrant (fig. 20 b), and the space a
at the crown of the vault is occupied by tracery commonly formed of one or more circles. The ribs are generally stopped at the circular boundary of the fan, and do not pass beyond it into this central tracery as they do in the larger examples (as fig. 24). But in these square plans the principle laid down at the beginning of the section, that the ribs are all strictly of the same curvature, must be held to apply only up to the altitude of the circular ring. There is generally no ridge rib introduced into the pattern of the tracery in A, and this space is either perfectly flat, as in the specimens already described at Gloucester and in the vault at All Souls (Pugin's Examples, vol. i.), or else is slightly domical, as in the vault of Hampton Court gateway (Pugin's Specimens, vol. ii.), so that in fact no ridge appears in the vault. When the domical form is carefully adjusted to that of the fans, so as to produce a continuity of curvature from the fans to the central space, the resulting vault is exceedingly rich and elegant; but when the fans abut against a flat ceiling, they are reduced in the decorative construction of the vault to mere corbels sustaining a slab.

In large specimens the square severy would increase the magnitude of the fans inordinately, and hence apparently the parallelogram was employed. But in such vaults the ribs are of the same curvature to the apex ; the ridge is marked by the same moulding as the other ribs, and is necessarily of the form given in fig. 20 (p.48), convex downwards. This imparts a certain heaviness of effect to the vault, which is partly redeemed by the exceeding richness of the tracery employed to decorate the surface, but which in my opinion places tiese compositions in architectural merit considerably below the complex lierne vaults which they superseded. The contrast in effect between the square plan and parallelogram for fan vaults may be seen at Peterborough, where some of the compartments of the vauit already quoted are of smaller span and square. In the table the square examples are marked (s.), and the parallelograms (p.).

A few fan vaults are constructed upon an apartment with a polygonal plan, as the Audley Chapel at Hereford, the Chantry Chapel in the cloisters at Westminster, the chapels of Urswick, Beaufort, and Lincoln, annexed to St. George's Chapel, Windsor, and the east end of Henry the Seventh's Chapel. The Audley Chapel at Hereford having a small span (thirteen feet six inches) and being semihexagonal, the fans are reduced to a small size with respect to the entire vault ; their radius is only two feet ten inches. They surround a central ring of masonry of about four feet radius, which is formed into a very flat dome, whose curvature unites but ciumsily with that of the fans. This vault being small, is wholly built of jointed masonry. The fans are formed of four stones each, one below and three above. The vault of Canterbury central tower is a
complex fan vault, having a fan in the centre of each wall as well as in the corners. There is a bell hole in the middle. This is a capital piece of workmanship. Its upper surface is accessible, and exhibits the flat surfaces of operation upon the ridge stones, but the masonry of the central portion is covered by a brick vault, apparently intended to stiffen it. The lower portions of the fans are of rib and pannel work as usual.

Vault of St. George's Chapel, Windsor.-This is a very singular vault. Fig. 30 (Plate I.) represents half the plan of one severy; the upper half of the plan shows the lower surface and the position of the joints with respect to the pattern, and the lower half of the plan shows the upper surface of the masonry. Fig. 31 is the section through DC. The portion of vault AFDE is a lierne vault with bosses of the ordinary kind, constructed entirely of rib and pannel work. The bosses, however, are not made with branches to receive the ribs, but like those of fig. 16 (p.27), in irregular polygons, including a portion of the vaulting surface. Their upper surfaces are left horizontal as usual. The crown of the vault, of which FEBC is one half, is of a totally different construction; it is built of solid jointed masonry in large slabs, and is merely a portion of a waggon vaalt of very obtuse angle, as shown in the section from $e$ to $c$. This difference of construction in the two portions of the vault, as weil as the level surfaces of operation of the boss stones, will be perhaps more easily seen in the isometrical projection fig. 32 (Plate IV.), which represents the apper surface of the vault, but is dissected at the sides and enci to show the relation of the upper and lower surfaces.

Mr. Rickman's description of this vault is so excellent that $I$ cannot do better than quote his words. "The roof of the nave and choir of St. George's, Windsor, is very singular and perhaps unique. The ordinary proportion of arches and piers is half the breadth of the nave; this makes the roof compartments two squares: but at Windsor the breadth of the nave is nearly three times that of the aisles, and this makes a figure of about three squares. The two exterior parts are such as, if joined, would make a very rich ribbed roof; and the central compartment, which runs as a flat arch, is filled with tracery pannels of various shapes, ornamented with quatrefoils, and forming two halves of a star; in the choir the centre of the star is a pendant. This roof is certainly the most singular, and perhaps the richest in effect of any we have; it is profusely adorned with bosses, shields, \&c." ${ }^{25}$

I may add, that the general form of the vault, like those of the choirs of

[^22]Gloucester and Wells, is a waggon vault, whose lateral or transverse vaulting cells enter it below its crown.

The contract for erecting the vault of the choir has been printed, amongst a great variety of interesting documentary notices relating to this building, by Ambrose Poynter, Esq. in his elaborate Essay on the History and Antiquities of Windsor Castle ${ }^{26}$. In this contract, dated 1505, J. Hylmer and W. Vertue agree, " in consideration of the sum of 700 l . to build with freestone the roof of the choir . . . containing seven severeys, in the same manner as the roof of the body. . . . Provided always that the principal keys of the said vault, from the high altar down to the king's stall, shall be wrought more pendaunt and hollower than the keys of the body of the chapel... and all the other lesser keys to be wrought more pendant and hollower than the keys in the body of the chapel, . . . all which works are to be finished at Christmas, 1508." It appears also from the same authority, that the vault of the nave had been recently completed at the time of making this contract, as the arms of Dean Urswick are displayed upon it.

The drawings in Plates I. and IV. were made from the vault of the choir, the upper surface of which happened to be accessible at the time of my visit. But as access could not be had to the central pendant below, I have omitted it in the section, consequently in this respect the drawing resembles the vault of the nave. This pendant is hollow, and complicated in structure. The central stone projects downwards very considerably from the surface of the vault, and receives the feet of twelve small ribs, which rise from it to meet the vault. The joints of the central portion of the vault are evidently disposed with careful regard to the pattern of the tracery; but as this is a waggon vault, and very nearly flat, the forms of the stones could bave been easily obtained from a plan of the actual surface, or perhaps was drawn upon a boarded centering.

[^23]
## Additional Remarks to Section II. on the Curvature of the Ribs.

Since writing the above paper, I have had an opportunity of examining the vaults of the cloister of Norwich Cathedral, which appear to me to furnish so excellent an illustration of the remarks in page 19, \&c. upon the attention paid by the Middle Age architects to the form of the spandrel solid, that I have thought it worth while to add the result of my observations.

The third volume of Britton's Antiquities contains an excellent description by Mr. Repton of this cloister, with ample illustrations ; and in the Norwich Cathedral (p. 38) are additional remarks, and a passage quoted from the Itinerary of William de Worcestre (pp. 302, 305), to which I beg to refer.

I shall merely remark that the cloister is the work of successive periods, but that the intention of the successive architects was evidently to make their continuations assimilate with the beginning; in which they have so well succeeded that at the first glance the entire work appears to be of one design.

This expression of uniformity is principally due to the employment of single simple shafts with base and capital in lieu of mullions throughout the tracery; for the pattern of the latter varies, and assumes in turn the peculiar fashions, geometrical, flowing, and perpendicular, that belong to the age in which each was erected. But the external arch which contains the tracery is of the same figure throughout.

I am informed by one of the canons residentiary, who has taken much pains in the investigation of matters relating to the structure of the cathedral, that the outer wall of the cloister is Norman, so that in all probability an early cloister (perhaps of wood) existed previously to the present one upon the same site, and was replaced piece by piece by the latter, as funds or convenience permitted.

The vaulting throughout is of the same plan, namely, each severy is nearly a square, there are transverse, diagonal, and wall ribs (the latter term also including the rib which lies next to the tracery), and one tierceron between each, so that the plan resembles that of the example in fig. 10 (p. 19), differing only in the proportions of the severey. The ridge ribs are horizontal. The curvature of the ribs, however, is different in the different periods, and it is to the variations in this respect that $I$ wish, in the present article, to direct attention.

The oldest vaults are the central severeys of the east walk, with which, according to W. de Worcestre, the work was commenced, in 1297; the south walk appears to be the next, and the whole of its vaults have the same character, but differ from the former. The west walk, which is also uniform in character, is the next in order; and the middle compartments of the north walk are, like the tracery which accompanies them, of decidedly Perpendicular style, and correspond to the period (1430) at which, the same author tells us, the cloister was completed. With respect to the extreme compartments of the east and north walks, the character is far less decided, and requires a farther investigation, which I have not upon this occasion had time to carry out. The account given by W. de Worcestre contains a history of the work, with the names of those who carried it on; but there are some difficulties in the application of these particulars, and he has only furnished the dates of the beginning and end of the work, as above quoted. I am inclined to think, from remaining appearances, that the vaults were in some parts left unfinished for a long time after the walls were completed, and that only the tas de charge was built, as we commonly find it even now in other buildings; for the curvatures of the ribs at the springing show traces of having been changed to accommodate the new fashions which had arisen when the ribs and vaults were added. For our present purpose, however, I shall select four examples, one out of the middle of each walk, the character of which is decided, and which will stand in order of time thus:-
(I) East, 1297.
(2) South.
(3) West.
(4) North, 1430.

The dimensions and form of the ribs in these four compartments were accurately measured.

The curvatures were abtained in the following manner. (Vide fig. B. p. 30.) A rod three feet in length had a plumb line suspended from its upper end. The lower end of this rod was placed upon the abacus and close to the springing of the rib whose curvature was to be measured, as at m. The upper end was brought into contact with the same rib as at $d$, so that $d \mathrm{D}$ represents the position of the plumb line. The horizontal distance mo (the versine of the arc $\mathrm{m} d$ ) was then measured, and thus the position of the point $d$ of the arch determined ${ }^{27}$. Evidently any number of points in the arch may be determined in the same manner, by applying a series of rods of known lengths, as one of four feet from $m$ to $c$, another of five feet from $m$ to $b$, and so on. But if the rib consist of a single arc of a circle, one such measurement is sufficient, provided

[^24]the height of the crown a $a$ can be measured, which is easily done, especially where the ridges of the vault are horizontal, and the altitudes of all the ribs consequently alike, as in this Norwich cloister. For the height of the wall rib is easily obtained, and serves for all. And since the spans of the ribs are all known from the plan, we have thus three points, $m, d, a$, given in position for each rib, from which, when laid down to scale upon the drawing board, the centre and radius of the arc can be found.

This method of measuring the curvatures has many advantages for travelling observations, for it requires merely a ladder to reach the impost, and can be carried on without help. Also a slight deviation from the horizontal position of the measuring rod md produces no appreciable error in the result. If on account of the distortion of the rib from settlement, which is always more or less to be expected in Middle Age work, the radius determined by this method should not be very accurate, it at least gives very precisely the middle plan of the spandrel from the very nature of the process. And besides, the point $d$ is contained in the tas de charge, which, being constructed of solid masonry, is not liable to suffer from settlement in a way to alter the curvature of the ribs, whereas any setclement of the shell of the vault changes the form of that portion of the ribs which lie above the solid masonry, so that we can only depend upon the tas de charge for giving is the original curvatures. But the spreading. of the walls and piers, by throwing the tas de charge out of the perpendicular, will of course affect the accuracy of the versine mo. To get rid of this source of error, I measured these versines at every one of the four corners of the severy in each of the examples, and took the mean of the four that belonged to each rib. For the most part, however, I found but little difference. However, for accurate results, observations should also be made with rods of two and four feet. A rod greater than five feet cannot be managed in this way without assistance. Care must be taken that the plumb line be suspended from that corner of the rod which is placed in contact, with the rib.

The following table contains the spans of the wall and transverse arches, and the altitudes of their crowns respectively; also the versines (or distances MD ) measured as above described, and the radius of each rib determined by the drawing board, upon which the points were laid down to a scale of one inch to the foot. The small figures which accompany the radii show the distance of each centre below the impost plane in inches and tenths.

The principal object of this table is to give the form of the middle plan of the spandrel solid, and the accuracy of this result may be relied on. The radii are of course only determined approximately, for want of a sufficient number
of points in the ribs. One obvious source of error in this case is the assumption that the ridge rib was intended to be horizontal, for as one of the points through which the arc passes is laid down upon the assumption that the height of all the ribs in the severy are alike, it is plain that if this were not the case the error would affect the radius. However, the radii here given will produce vaults having as nearly as possible the same form of the spandrel solid as those from which the measurement was taken, which appears to me to be the most important point. In fact, at present the crowns of the vaults in the eastern walk have sunk about three inches, and thus disturbed the horizontality of the ridge; but the form of ridge so produced is so irregular and so different in the different compartments, that it is evidently the result either of bad workmanship or of settlement, and not of design. On the other hand, in the west walk, the crowns of the vaults are slightly raised, and this I suppose to be intended to allow for the effect of a similar settlement, for it is not considerable enough to be intended to give a domical form. These circumstances have been duly allowed for in determining the radii. The transverse, tierceron, and diagonal arches of the north walk are, however, four-centred, and I was unable for want of time to determine with proper accuracy the radii of their upper circles or the proportion of the two.


Fig. 33 shows the form of the middle plans of the spandrels in the four selected examples, obtained from the curvature of the ribs by a construction similar to that of fig. 10 (p. 19). In the east walk, marked E, A b AD are the diagonal ribs, at the transverse rib, a w aw the wall ribs, and the inter-
mediate lines the tiercerons. The dots at the extremities of these lines show the points at which the fillets of the ribs pass through this middle plan, and it will be seen that in this case the points WDDW form a rectangle or double square, and that $T$ is on the side of this rectangle; but the tiercerons retire slightly within it. The appearance


Fig. 33. of this spandrel solid is that of a mass decidedly rectangular in its horizontal sections, but the angles of which are rendered more prominent by the retiring inwards of the tiercerons. $S$ shows the middle plan for the south walk, to which no letters of reference are added, as it is in general system exactly like the preceding. In this vault the diagonal, wall, and transverse ribs still form a square mass, but the tiercerons advance beyond this mass instead of retiring. within it, thus giving an imperfectly polygonal character to the whole, or rather constituting a double polygon with unequal angles.

In the middle plan for the west walk, W, the diagonal rib retires within the rectangular form, and the transverse advances beyond it, so as to give a completely polygonal figure to the mass in its horizontal sections, but yet to make a polygon with unequal angles. This is a very favourite section for vaults of this period. But in the north walk the ribs (with the exception of the wall ribj are semi feur-centred arches, having lower circles of the same radius, and consequently the middle plan N (taken through these lower circles), is an equiangular polygon. But as the curvature of the wall rib is greater than that of the others, its position in this section is thrown nearer to the centre, and this, by causing the vaulting surface to meet the wall at an acute angle, relieves and detaches the entire spandrel from the wall in a very bold and striking manner. The same effect will be observed in the two previous examples $S$ and $W$.

It thus appears that the form of the middle plan changes by gradual su:cession from a square to a polygon; and as the three first examples consist of ribs of a single arc, these changes of form are all brought about by varying and altering the radii and the positions of the centres of the respective ribs.

Similar forms to all these may be found in cotemporary vaults, but it rarely happens that they occur in a series and in a work of which the general uniformity of design is manifestly preserved, so that the changes may be regarded in the light of embellishments or as the modern improvements of their day. In this respect the Norwich cloister is exceedingly valuable, by enabling us to
discover many of these improvements which it is more difficult to pick out from examples complete in the character of their own age.

A walk round the cloister will at once show, by bringing the various forms into direct comparison, how much influence the form of the spandrel exerts upon the character of the work ; and upon considering the various methods by which the different radii and curvature of ribs in a vault might have been determined, and the probable motives for the different forms they are found to have assumed, I cannot discover any that have a greater appearance of probability than that of giving a certain form to the spandrel solid; for the shape of this, as determined by its horizontal sections, is so obtrusive a characteristic of every vault, and the variations of it appear to succeed each other so regularly, that I cannot suppose them to be accidental. Besides, the motive thus supplied for changing and adjusting the curvatures is of so practical a character and produces such tangible results, that it is much more likely to have been the true one than any occult numerical or geometrical relations between the radii of the several ribs. How easy it is, for example, in looking at any given vault with a view to copy it, to observe that its diagonal rib is too prominent, and therefore requires its centre to be thrown higher up or its curvature increased in order to make its middle points retire inwards, and this is precisely what has happened in the successive compartments of Norwich cloister.

Analogous changes of form constitute in effect the materials for a history of architectural decoration in general. Whateve: propriety or beauty a given form may possess, we find that, by long continued use and mnltiplication, it palls upon the eye and requires change. The very characteristic by which it pleased the most, and which was nurtured and developed by every possible adjunct, will be in the next age softened down and subdued to make way for the prominence of some other, which was once in the same manner avoided or made subordinate. So in the series of spandrels we are now considering, the square section at first pleased the eye, and was rendered more prominent by the retirement of the tiercerons; but in the succeeding specimens this effect is gradually softened down until we obtain an equiangular polygon. But there are also mechanical reasons, as well as those derived from beauty of form, that may account for the growing preference for the polygonal section; for in the latter forms (as also in the fan vaults) the horizontal sections constitute arches that oppose the rising of the haunches of the ribs, and thus the ribs mutually stiffen each other through the intervening surfaces. This is not the case in vaults of a square section, in which the transverse rib receives no such assistance from. the diagonal. It would lead me, however, far beyond the proper limits of this
paper were I to enter into mechanical considerations, which I have hitherto avoided.

I have said that the spandrel is detached from the wall by so managing the curvatures of the wall rib and first tierceron that the vaulting surface may meet the wall at an acute angle. The beauty and effect of this may be understood by looking at the opposite effect in the vault of the cathedral. This is a lierne vault of elaborate pattern, but of very rough workmanship. The curvature of the ribs is rude and broken, and displays so little management that I am tempted to believe the vault to have been wrought out by country masons with very little or no geometrical knowledge or system. The wall rib, instead of rising upon stilts and retiring, as in most clerestory vaults, for the purpose of making way for the window, is, on the contrary, an arch of a lower crown than the ridge of the vault, and is very straight-sided and misshapen in form. The transverse ridge rib rises considerably in the middle, and is curved so as to form a real segmental arch, and the middle plan of the spandrel is nearly a semicircle. The effect of these arrangements is that the spandrel seems to adhere timidly to the wall instead of springing boldly across the space, as in the vaults of the cloister.

The vaults of St. Mary Redcliffe, at Bristol, especially those of the north transept, are very striking examples of the lightness and boldness of character that may be given to a vault by setting back the vaulting surface where it meets the walls.

In the diagrams, fig. 10, p. 19, and others of the section on curvature, the ribs are represented by mere lines, which lines are actually the lower fillets of the ribs. Thus all that has been said of the middle section of the solid must be understood of an imaginary surface stretched from the front of each rib to its neighbouring rib, which is in truth the actual effect produced by the group of ribs upon the eye. And the horizontal section may be easily drawn when the curvatures are given, by means of fig .10 . This diagram, as explained in its place, is intended to give the curvatures after the middle plan has been drawn, but will easily furnish the reverse process; for let the plan of one of these Norwich vaults, for example, be drawn, and the elevations of the ribs inserted as in the figure by help of the table (p.64) of radii; then to obtain the section of the solid at any required height $\mathrm{R} s$, we have only to set off points $s, t, m, b, p$, at the same vertical height upon every rib, and drop perpendiculars from them upon the respective plan lines, intersecting the latter in $\mathrm{r}, r, w, a, z$. Lines joining these points will give the desired form of the middle section. By drawing in this manner a series of equidistant horizontal sections from the impost to the
crown of the vault, the resulting lines upon the plan will form a species of shading that will show the exact nature of the entire form, much in the same manner as the topographical shading of mountains and broken ground in a survey is made to represent their exact solid figure upon the mere plan ${ }^{26}$.

I have employed this system in sketching the plans of vaults, for by shading them in the intermediate compartments between the ribs with lines disposed according to this method as nearly as the eye will determine, the general disposition of the surfaces may be recorded with sufficient accuracy, and such a record is very desirable according to the views I have endeavoured to advance of the importance and effect of these dispositions upon the æsthetic character of the vault.

In conclusion, I must beg it to be recollected that I offer this paper as a sketch of an investigation, for the completion of which I request the assistance of the members of the Institute in the collection of facts and examples. I have endeavoured throughout to show from evidence the existence and employment of geometrical methods from a very early period, and have attempted to restore some of these methods. I have also ventured to assert the importance of certain forms and arrangements in imparting character to the buildings in question. The necessary limits of a paper of this kind have prevented me from introducing several topics which may appear to belong to the question in hand. Thus I have said nothing respecting mechanical principles, and have confined myself to form and arrangement. But it appears to me, from examination of

[^25]the works of the Middle Age architects, that the latter considerations had an infinitely greater influence upon their structures than the relations of pressure, then very little understood, and about which they made manifest and sometimes fatal errors ; so that this omission may be fairly allowed, or at least the discussion of this part of the subject may be carried on separately. Also with respect to the elaborate and beautiful class of vaults which I have termed Lierne Vaults, I propose, at some future time, to offer to the Institute a supplementary paper containing many observations upon them, and probably additional remarks upon the entire subject. Lastly, since the methods which I have explained in this paper have been for the most part elicited from an examination of the existing vaults, it may be worth while to compare them with those that are recommended and taught in the books of stone-cutting already quoted and referred to, as well as with modern actual practice.

# TRANSACTIONS 

OF THE

# ROYAL INSTITUTE OF BRITISH ARCHITECTS 

of

## LONDON.

VOL. I. PARTII.

USUI CIVIUM, DECORI URBIUM.

LONDON:
LONGMAN, BROWN, GREEN, AND LONGMANS.
1842.


[^0]:    ${ }^{1}$ Of plain Norman waggon vaults, there are two excellent specimens at Norwich, one of them, under the Bishop's Palace, of about twenty feet span, supported at short intervals by subribs, plain and square edged, and resting upon semicircular plain corbels. A string cornice of the usual Norman profile runs along the impost line of the entire apartment, and is mitred round the corbels. The other specimen is an apartment on the south side of the nave of the cathedral at the west end, once probably the guests' hall, now the kitchen of a prebendal house.
    ${ }^{2}$ Vide an excellent paper by De Lassaux, on a mode of erecting light vaults. Journal of Royal Institution, vol. i. p. 224.

[^1]:    ${ }^{3}$ Accurate plans and sections, with details of this vault and of the castle, are in the Vetusta Monumenta. There are also some rough sketches in Carter's Ancient Architecture.

[^2]:    *This block of masonry appears to have been termed the tas de charge, for Philibert de l'Orme㤢伿s this term thus:-"Ce sont les premieres pierres que on voit sur les angles, et monstrent le commencement et la naissance des branches, des ogives, tiercerons, formerets et arcs doubleaux." These live terms are the names of the ribs, as we shall see presently.
    ${ }^{5}$ I employ the term rib and pannel work to distinguish that mechanical construction of the vaults Wif the Middle Ages, in which a frame work of ribs is made to support thin superincumbent vaults in manner of pannels, from the vaults of solid masonry which were subsequently introduced, in which
    She stones are closely jointed throughout, and the ribs and pannels merely carved on their lower deco-
    Wive surfaces. This latter system, which is in fact derived from the original Roman or earlier vaults,
    Wholly adopted in Henry the Seventh's Chapel at Westminster. But the two methods are, as will mpear in the course of this paper, mixed together in the greater number of examples.

[^3]:    ${ }^{6}$ Vide the essay of De Lassaux.

[^4]:    ${ }^{7}$ Tracts on Vaults and Bridges.

[^5]:    ${ }^{8}$ Architecture of the Middle Ages, p. 72.

[^6]:    9 The ridges of fan vaults are convex downwards, but they belong to the class of vaults with fourcentered ribs.

[^7]:    ${ }^{10}$ The arches in this example are very highly pointed; the compartment is sixteen feet by eighteen feet two inches, and the height of the crown of the vault seventeen feet seven inches, or about eighteen feet if measured to the vaulting surface. The height is, therefore, the same as that of the long side of the parallelogram, which perhaps was the principle that in this case determined the proportions.

[^8]:    ${ }^{11}$ These arches of Hereford north transept are so excessively straight-sided, that I measured them carefully, and with the following results. The height в с (fig. 9.a) is seven feet eight inches; the span is eleven feet six inches (so that the height is two-thirds of the span). The versed sine, or distance of the middle point $E$ of the chord $A C$ from the arch, is seven inches and five-eighths. The radius obtained from these measures, either by calculation or by laying down the arch to scale, comes out eighteen feet five inches, and it also appears that the vertical distance of the centre of the segment ac below the impost line is very nearly equal to the altitude $\boldsymbol{B}$ c of the crown of the arch above that line.

[^9]:    ${ }^{12}$ According to Philibert de l'Orme, the simple geometrical construction by which an arc of a circle is described through three given points (not in one straight line), is strangely termed "Manière de trouver les trois poinctz perdus." The curved templet or wooden sweep which is obtained from this process is called the "cherche ralongée."-" Inventions pour bien bastir," p. 21.

    The fact of this problem having a familiar name shows it to have been in common use amongst workmen. Our workmen similarly term the mode of setting out a rectangle (from Euc. i. 47) the rule of six, eight, and ten, these being the convenient whole numbers of feet that possess the required property, namely, that the sum of the squares of the two first shall equal the square of the last, or $36+64=100$. This rule is given in Vitruvius (b. ix. c. 2) from Pythagoras, and the numbers of feet used are three, four, and five.

[^10]:    ${ }^{13}$ He flourished from 1536 to 1577. The contract for King's College vault is dated 1513, and the first stone of St. Peter's at Rome was laid in 1506.
    ${ }^{14}$ Traité de Stereotomie, t. iii. p. 23, 3rd edit. 1769.
    ${ }^{15}$ Art of Sound Building, 1725.
    ${ }^{16}$ Builder's Director, new edit. 1834, p. 79.
    ${ }_{17}$ Treatise on Masonry and Stonecutting, 1828, p. 91.

[^11]:    ${ }^{18}$ The centres are easily found in each case, as for $\mathrm{E} f$ by joining $\mathrm{E} f$, bisecting and drawing a pero pendicular, which will intersect the line $E H$ in the required centre $F$.

[^12]:    ${ }^{17}$ By the middle plane is here meant a vertical plane seated upon the plan line of any rib (as AD), and which plainly divides such rib into two similar and opposite halves.

[^13]:    ${ }^{18}$ The plan of the vault from which this boss was taken was, however, much more complicated than that which I have given in fig. 13 , but I have taken care to place the boss $m$ in precisely the same

[^14]:    19 Architecture of the Middle Ages, especially of Italy, p. 23.

[^15]:    ${ }^{20}$ There is a plan and view of this vault in Britton's Wells Cathedral.

[^16]:    ${ }^{21}$ For the details of King's College vault I shall refer to the accurate drawings of Mr. Mackenzie, in the new edition of Tredgold's Carpentry.

[^17]:    ${ }^{22}$ These beds are in fact conical surfaces, which may be obtained theoretically. Draw az perpendicular to AB , then will $\mathrm{A} Z$ be the axis of the solid of revolution or spandrel-conoid generated by the

[^18]:    ${ }^{23}$ Sometimes, when the pattern of the vault admits of it, one or both of the ridge joints $d y t w$ are omitted, and the stone brought to a point on the plan at $t w$ or $d y$. This happens to be the case in the stone which at Peterborough corresponds to the one I am now describing, as will be seen in fig. 24; but as the two joints in question are very often employed $I$ have inserted them in this place to make the description of the method complete.

[^19]:    ${ }^{24}$ This is not well exhibited in the woodcut. The plain sides of the lierne $c d$ should be of equal height, and the vaulting surface raised on the right hand, and dropped on the left accordingly. The sketch Ab was made from one of the liernes of the vault of Henry the Fifth's tomb at Westminster.

[^20]:    ${ }^{25}$ Chandler's Life of Waynflete, p. 140.

[^21]:    ${ }^{26}$ The plan both of these vaults and of the chapels just described may be understood from Carter's large plan, published by the Society of Antiquaries, and may also be traced, but with some difficulty, on account of the small scale, in Britton's Gloucester.

    The west front, and consequently the porch in question, has been attributed to Abbot Morwent (1420—1437).

[^22]:    ${ }^{25}$ Rickman's Attempt to discriminate, p. 104.

[^23]:    ${ }^{26}$ Prefixed to "Illustrations of Windsor Castle," by the late Sir Jeffry Wyatville, R.A. 1841. In this Essay, the date 1528 is assigned to the fan vault, of the central compartment or crossing, and 1537 to the fan vaults of the side aisles of the choir upon heraldic evidence.

[^24]:    ${ }^{27}$ For $d$ is the apex of a right-angled triangle, of which the hypothenuse $m$ is known, and also one side, MD , therefore the triangle is given.

[^25]:    ${ }^{26}$ If this method be followed out, it is capable of great precision. When the ribs are represented by single lines, the form of the entire group will be shown, as above. But the sections of the vaulting surfaces are often different from those of the group of ribs at the same level, and the sections of these surfaces may be obtained as follows. The ribs in the plan must be represented by parallel lines at the distance corresponding to the thickness of the rib, as in fig. 16 (p.27), and not by single lines. The curves employed to obtain the middle plans must be the backs of the ribs, and not their front fillets. These, being the lines of junction between the vaulting surfaces and the ribs, are concentric to the front fillets for the most part. It is unnecessary to draw sections sufficiently close to shade the vault, as eight or ten between the crown and impost are sufficient to show the form of the vault, but in such a drawing care must be taken to note that these lines are not intended for joint lines.

    The elevation of the curves must all be drawn with reference to a single base line, as in fig. 16, and not placed each on its own plan line, as in fig. 10. For thus a series of lines drawn parallel to the base G $\nless$ (fig. 16) through all the ribs, will represent the parallel sections. A vertical line or axis upon the impost $G$ will intersect all these, and the distance from this axis of the successive points of intersection of any given rib as it passes through the parallel sections in order, may be marked upon the edge of a slip of paper, and then transferred to the plan line of the rib in question. When this has been effected for each rib, the corresponding points must be joined, and thus the sections marked upon the plan. I find this process easy and rapid in practice, although I fear it will be scarcely intelligible without a figure. In fig. 15 (Plate I.) the left hand half is shaded upon this system, and the right hand half has parallel sections in dotted lines. These were sketched in, however, by eye merely.

